EECS 442 – Computer vision

Single view metrology

- Review calibration
- Lines and planes at infinity
- Absolute conic
- Estimating geometry from a single image
- Extensions

Reading: [HZ] Chapters 2, 3, 8
Calibration Problem

\[ \mathbf{M} = \mathbf{K}[\mathbf{R} \quad \mathbf{T}] \]

\[
\begin{bmatrix}
\alpha & -\alpha \cot \theta & u_o \\
0 & \beta & v_o \\
0 & \sin \theta & 1
\end{bmatrix}
\]

\[ \mathbf{P}_i \rightarrow \mathbf{M} \mathbf{P}_i \rightarrow \mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \]

World ref. system  
In pixels
Calibration Problem

\[ \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} \]

World ref. system \hspace{2cm} In pixels

\[ \mathbf{M} = \mathbf{K}[\mathbf{R} \hspace{1cm} \mathbf{T}] \]

11 unknown

Need at least 6 correspondences
Once the camera is calibrated...

\[ \mathbf{M} = \mathbf{K}[\mathbf{R} \quad \mathbf{T}] \]

- Internal parameters $\mathbf{K}$ are known
- $\mathbf{R}$, $\mathbf{T}$ are known – but these can only relate $\mathbf{C}$ to the calibration rig

Can I estimate $\mathbf{P}$ from the measurement $\mathbf{p}$ from a single image?

No - in general $\otimes$ [$\mathbf{P}$ can be anywhere along the line defined by $\mathbf{C}$ and $\mathbf{p}$]
Recovering structure from a single view

unknown

known

Known/Partially known/unknown
Recovering structure from a single view

http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl
• Review calibration
• Lines and planes at infinity
• Absolute conic
• Estimating geometry from a single image
• Examples
Lines in a 2D plane

$ax + by + c = 0$

$l = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

If $x = [x_1, x_2]^T \in l$

\[
\begin{bmatrix} x_1^T \\ x_2 \\ 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0
\]
Lines in a 2D plane

Intersecting lines

\[ x = 1 \times 1' \]

Proof

\[ 1 \times 1' \perp 1 \quad \rightarrow \quad (1 \times 1') \cdot 1 = 0 \quad \rightarrow \quad x \in l \]

\[ 1 \times 1' \perp 1' \quad \rightarrow \quad (1 \times 1') \cdot 1' = 0 \quad \rightarrow \quad x \in l' \]

\[ x \]

\[ \rightarrow x \text{ is the intersecting point} \]
Points and planes in 3D

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}, \quad \Pi = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \]

\[ x \in \Pi \iff x^T\Pi = 0 \quad \Rightarrow \quad ax + by + cz + d = 0 \]

How about lines in 3D?

- Lines have 4 degrees of freedom - hard to represent in 3D-space
- Can be defined as intersection of 2 planes
Points at infinity (ideal points)

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad x_3 \neq 0 \]

\[ x_{\infty} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \]

Let's intersect two parallel lines:

\[ \rightarrow l \times l' = (c - c') \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} \]

Agree with the general idea of two lines intersecting at infinity
Lines infinity $1_{\infty}$

Set of ideal points lies on a line called the line at infinity
How does it look like?

Indeed:

$$1_{\infty} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$
Projective transformation of a line (in 2D)

\[ H = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \]

\[ l' = H^{-T} l \]

\[ H^{-T} l_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ b \end{bmatrix} \quad \text{...no!} \]

\[ H_A^{-T} l_\infty = ? = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A^{-T} \\ -t^T A^{-T} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
Projective transformation of a line (in 2D)

- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
  - if yes, these 2 lines are // in 3D

\[ l_{\text{hor}} = H^{-T} l_{\infty} \]

- Recognition helps reconstruction!
- Humans have learnt this
Vanishing points  (= ideal points in 2D)

Points where parallel lines intersect in 3D
Vanishing points and their image

direction of the line

\[ \mathbf{v} = \mathbf{K} \mathbf{d} \]

\[ \mathbf{M} = \mathbf{K}[\mathbf{R} \quad \mathbf{T}] \]
Vanishing points - example

v1, v2: measurements
K = known and constant

Can I compute R?

\[ d_1 = \frac{K^{-1} v_1}{\|K^{-1} v_1\|} \]

\[ d_2 = \frac{K^{-1} v_2}{\|K^{-1} v_2\|} \]

\[ d_1 = R \ d_2 \rightarrow R \]
\[ d_1 = \frac{K^{-1} v_1}{\|K^{-1} v_1\|} \]
\[ d_2 = \frac{K^{-1} v_2}{\|K^{-1} v_2\|} \]
Planes at infinity & vanishing lines

\[ \Pi_\infty = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \]

- Parallel planes intersect at the plane at infinity
- 2 planes are parallel iff their intersections is a line that belongs to \( \Pi_\infty \)
- Parallel planes intersect the plane at infinity in a common line – the vanishing line (horizon)
Vanishing lines and their images

Parallel planes intersect the plane at infinity in a common line – the vanishing line (horizon)

\[ n = K^T l_{\text{horiz}} \]
• Review calibration
• Lines and planes at infinity
• **Absolute conic**
• Estimating geometry from a single image
• Examples
Conics in 2D

\[ ax^2 + bxy + cy^2 + dx + ey + f = 0 \]

In homogeneous coordinates:

\[ x^T C x = 0 \]

\[ C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \]

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \]

- If a point \( x \in C \) \( \rightarrow x^T C x = 0 \)
- If a line \( l \) is tangent to a conic in \( x \) \( \rightarrow l = C x \)
- Projective transformation of conics:

\[ C' = P^{-T} C P^{-1} \]
Circular points

Circular points are \textit{fixed} under similarity transformation

\[
H_s p_i = \begin{bmatrix}
 s \cos \theta & -s \sin \theta & t_x \\
 s \sin \theta & s \cos \theta & t_y \\
 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
 1 \\
 i \\
 0
\end{bmatrix} = \begin{bmatrix}
 1 \\
 i \\
 0
\end{bmatrix}
\]

Circular points

(point at infinity)

1

1_{\infty}

\[i^2 = -1\]
Circular points define a degenerate conic called $C_\infty$; any $x \in C_\infty$

\[
x^T C_\infty x = 0 \quad C_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
\begin{cases} x_1^2 + x_2^2 = 0 \\ x_3 = 0 \end{cases}
\]

$C_\infty$ is fixed under similarity transformation
In 3D: absolute conic $\Omega_\infty$ is a $C \in \Pi_\infty$

Any $x \in \Omega_\infty$ satisfies:

$$x^T \Omega_\infty x = 0 \quad \Omega_\infty = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{cases} x_1^2 + x_2^2 + x_3^2 = 0 \\ x_4 = 0 \end{cases}$$

$\Omega_\infty$ is fixed under similarity transformation
Projective transformation of $\Omega_\infty$

$$\omega = (K^T K)^{-1}$$

$$P = K\begin{bmatrix} R & T \end{bmatrix}$$

1. It is not function of $R, T$

2. $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$ symmetric

3. $\omega_2 = 0$ zero-skew

4. $\omega_1 = \omega_3$ square pixel
Projective transformation of $\Omega_\infty$

$$\omega = (K^T K)^{-1}$$

Why this is useful?
• Review calibration
• Lines and planes at infinity
• Absolute conic
• Estimating geometry from a single image
• Examples
Angle between 2 vanishing points

\[
\cos \theta = \frac{v_1^T \omega \ v_2}{\sqrt{v_1^T \omega \ v_1} \ \sqrt{v_2^T \omega \ v_2}}
\]

If \( \theta = 90 \) \( \rightarrow \) \( v_1^T \omega \ v_2 = 0 \)
Angle between 2 scene lines

\[ \theta = 90 \]

\[
\begin{cases}
    v_1^T \omega v_2 = 0 \\
    \omega = (K^T K)^{-1}
\end{cases}
\]

Constraint on K
Single view calibration - example

\[ \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & \omega_6 \end{bmatrix} \]

Compute \( \omega \):
- 6 unknown
- 5 constraints
- \( \omega \) known up to scale

\[ \omega = (K^T K)^{-1} \]

\( \rightarrow K \) (Cholesky factorization)
Single view reconstruction - example

\[ K \text{ known} \rightarrow n = K^T l_{\text{horiz}} \]

= Scene plane orientation in the camera reference system

Select orientation discontinuities
Single view reconstruction - example

Recover the structure within the camera reference system

Notice: the actual scale of the scene is NOT recovered

- Recognition helps reconstruction!
- Humans have learnt this

Are these two lines parallel or not?
- Recognize the horizon line
- Measure if the 2 lines meet at the horizon
- If yes, these 2 lines are // in 3D
http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl
La Trinita' (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)
La Trinita (1426)
Firenze, Santa Maria Novella; by Masaccio (1401-1428)
Manually select:
• Vanishing points and lines;
• Planar surfaces;
• Occluding boundaries;
• Etc..
Automatic Photo Pop-up

Hoiem et al, 05
Automatic Photo Pop-up

Hoiem et al, 05…
Automatic Photo Pop-up

Hoiem et al, 05...

Software:

Make3D

Saxena, Sun, Ng, 05…

Plane Parameter MRF

\[ P(\alpha | X, \nu, y, R; \theta) = \frac{1}{Z} \prod_i f_1(\alpha_i | X_i, \nu_i, R_i; \theta) \prod_{i,j} f_2(\alpha_i, \alpha_j | y_{ij}, R_i, R_j) \]

(a) Connectivity

(b) Co-Planarity

youtube
A software: **Make3D**

"Convert your image into 3d model"  http://make3d.stanford.edu/

http://make3d.stanford.edu/images/view3D/185
http://make3d.stanford.edu/images/view3D/931?noforward=true
Coherent object detection and scene layout estimation from a single image

Y. Bao, M. Sun, S. Savarese, CVPR 2010, BMVC 2010
Next lecture:

Multi-view geometry (epipolar geometry)