EECS 442 – Computer vision

Stereo systems

- Stereo vision
- Rectification
- Correspondence problem
- Active stereo vision systems

Reading:  [HZ] Chapter: 11
[FP] Chapter: 11
Goal: estimate the position of P given the observation of P from two view points

Assumptions: known camera parameters and position (K, R, T)
Stereo vision

Subgoals:
- Solve the correspondence problem
- Use corresponding observations to triangulate
• Given a point in 3d, discover corresponding observations in left and right images
Intersection of the two lines of sight gives rise to $P$
Parallel image planes

- When views are **parallel** these two steps becomes much easier!
Epipolar geometry

- Epipolar Plane
- Baseline
- Epipolar Lines

- Epipoles $e_1, e_2$
  - intersections of baseline with image planes
  - projections of the other camera center
  - vanishing points of camera motion direction
Parallel image planes

- Parallel epipolar lines
- Epipoles at infinity
- $v = v'$

Rectification: making two images “parallel”
Parallel image planes

$K_1 = K_2 = \text{known}$

$x \parallel O_1O_2$

$E = [t_x]R$
Cross product as matrix multiplication

\[
a \times b = \begin{bmatrix}
0 & -a_z & a_y \\
a_z & 0 & -a_x \\
-a_y & a_x & 0
\end{bmatrix}\begin{bmatrix}
b_x \\
b_y \\
b_z
\end{bmatrix} = [a \times]b
\]
Parallel image planes

$K_1 = K_2 = \text{known}$

$x$ parallel to $O_1O_2$

\[
E = [t_x]R = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -T \\
0 & T & 0
\end{bmatrix}
\]

$\rightarrow v = v'$?
Parallel image planes

\[
p^T E p' = 0
\]

\[
\begin{pmatrix}
u \\
v \\
1
\end{pmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & -T & \\
0 & T & 0 & 1
\end{bmatrix}
\begin{pmatrix}
u' \\
v'
\end{pmatrix} = 0
\]

\[
\begin{pmatrix}
u \\
v \\
1
\end{pmatrix}
\begin{bmatrix}
0 \\
-T \\
Tv'
\end{pmatrix} = 0
\]

\[
Tv = Tv'
\]
Making image planes parallel

GOAL: Estimate the perspective transformation $H$ that makes the images parallel
Projective transformation

Now we don’t have the destination image 😞
Making image planes parallel

**GOAL:** Estimate the perspective transformation $H$ that makes images parallel

- **Impose $v'=v$**

  - This leaves degrees of freedom for determining $H$
  - If not appropriate $H$ is chosen, severe projective distortions on image take place
  - We impose a number of restriction while computing $H$
Making image planes parallel

0. Compute epipoles

\[
e = KR^T T = [e_1 \quad e_2 \quad 1]^T
\]

\[
e' = K' T
\]
Making image planes parallel

1. Map e to the x-axis at location $[1,0,1]^T$ (normalization)

$$e = [e_1 \ e_2 \ 1]^T \rightarrow [1 \ 0 \ 1]^T$$

$$H_1 = R_HT_H$$
Making image planes parallel

2. Send epipole to infinity:

\[ e = [1 \ 0 \ 1]^T \rightarrow [1 \ 0 \ 0]^T \]

\[
H_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-1 & 0 & 1 \\
\end{bmatrix}
\]

Minimizes the distortion in a neighborhood (approximates id. mapping)
Making image planes parallel

3. Define: $H = H_2 H_1$

4. Align epipolar lines
Projective transformation of a line (in 2D)

\[ H = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \]

\[ l \rightarrow H^{-T} l \]
Making image planes parallel

3. Define: $H = H_2 H_1$

4. Align epipolar lines

$H'^{-T} l' = H^{-T} l$

These are called **matched pair** of transformation

[HZ] Chapters: 11 (sec. 11.12)
Making image planes parallel

Courtesy figure S. Lazebnik
Why rectification is useful?

- Makes the correspondence problem easier
- Makes triangulation easy
Application: view morphing

Morphing without using geometry
Rectification
Stereo vision

Subgoals:
- Solve the correspondence problem
- Use corresponding observations to triangulate
Computing depth
Computing depth

Note: Disparity is inversely proportional to depth

\[ x - x' = \frac{B \cdot f}{z} = \text{disparity} \]
Disparity maps

\[ x - x' = \frac{B \cdot f}{z} \]

Stereo pair

Disparity map / depth map

Disparity map with occlusions

http://vision.middlebury.edu/stereo/
Stereo vision

Subgoals:
- Solve the correspondence problem
- Use corresponding observations to triangulate
Correspondence problem

Given a point in 3d, discover corresponding observations in left and right images [also called binocular fusion problem]
Correspondence problem

• A Cooperative Model (Marr and Poggio, 1976)

• Correlation Methods (1970--)

• Multi-Scale Edge Matching (Marr, Poggio and Grimson, 1979-81)

[FP] Chapters: 11
Correlation Methods (1970--)

- Pick up a window around $p(u,v)$
• Pick up a window around \( p(u,v) \)
• Build vector \( W \)
• Slide the window along \( v \) line in image 2 and compute \( w' \)
• Keep sliding until \( w \cdot w' \) is maximized.
Normalized Correlation; minimize:

$$\frac{(w - \bar{w})(w' - \bar{w}')}{{\| (w - \bar{w})(w' - \bar{w}') \|}}$$
Correlation methods

Credit slide S. Lazebnik
Correlation methods

- Smaller window
  - More detail
  - More noise

- Larger window
  - Smoother disparity maps
  - Less prone to noise

Window size = 3

Window size = 20

Credit slide S. Lazebnik
Issues

• Fore shortening effect

• Occlusions

It is desirable to have small B/z ratio!
Issues

- small $B/z$ ratio

Small error in measurements implies large error in estimating depth
Issues

• Homogeneous regions

Hard to match pixels in these regions
Issues

- Repetitive patterns
Correspondence problem is difficult!

- Occlusions
- Fore shortening
- Baseline trade-off
- Homogeneous regions
- Repetitive patterns

Apply non-local constraints to help enforce the correspondences
Results with window search

Data

Ground truth

Window-based matching

Credit slide S. Lazebnik
Improving correspondence: Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
Improving correspondence: Non-local constraints

- **Uniqueness**
  - For any point in one image, there should be at most one matching point in the other image

- **Ordering**
  - Corresponding points should be in the same order in both views

Not always in presence of occlusions!
Dynamic Programming (Baker and Binford, 1981)

[Uses ordering constraint]

- Nodes = matched feature points (e.g., edge points).
- Arcs = matched intervals along the epipolar lines.
- Arc cost = discrepancy between intervals.

Find the minimum-cost path going monotonically down and right from the top-left corner of the graph to its bottom-right corner.

courtesy slide to J. Ponce
Dynamic Programming (Baker and Binford, 1981)

for \( k = 1 \) to \( m \) do
  for \( l = 1 \) to \( n \) do
    \% Initialize optimal cost \( C(k, l) \) and backward pointer \( B(k, l) \).
    \( C(k, l) \leftarrow +\infty; B(k, l) \leftarrow \text{nil}; \)
    \% Loop over all inferior neighbors \((i, j)\) of \((k, l)\).
    for \((i, j)\in\text{Inferior - Neighbors}(k, l)\) do
      \% Compute new path cost and update backward pointer if necessary.
      \( d \leftarrow C(i, j) + \text{ArcCost}(i, j, k, l); \)
      if \( d < C(k, l) \) then \( C(k, l) \leftarrow d; B(k, l) \leftarrow (i, j) \) endif;
      endfor;
    endfor;
  endfor;
% Construct optimal path by following backward pointers from \((m, n)\).
\( P \leftarrow \{(m, n)\}; (i, j) \leftarrow (m, n); \)
while \( B(i, j) \neq \text{nil} \) do \( (i, j) \leftarrow B(i, j); P \leftarrow \{(i, j)\} \cup P \) endwhile.
Dynamic Programming (Ohta and Kanade, 1985)

Improving correspondence: Non-local constraints

• Uniqueness
  – For any point in one image, there should be at most one matching point in the other image

• Ordering
  – Corresponding points should be in the same order in both views

• Smoothness
  – Disparity is typically a smooth function of x (expect in occluding boundaries)
Smoothness
Stereo matching as energy minimization

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 01

\[ E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D) \]

\[ E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2 \]

\[ E_{\text{smooth}} = \sum_{\text{neighbors } i,j} \rho(D(i) - D(j)) \]

• Energy functions of this form can be minimized using graph cuts
Stereo matching as energy minimization

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 01

Ground truth

Window-based matching

Graph cuts
Two-frame stereo correspondence algorithms

Click here

http://www.middlebury.edu/stereo/
Stereo SDK stereo vision software development kit.

A. Criminisi, A. Blake and D. Robertson

Demo video:
Real-time dense stereo matching

http://research.microsoft.com/vision/cambridge/i2i/MSRC_SDK
Application – foreground/background separation

V. Kolmogorov, A. Criminisi, A. Blake, G. Cross and C. Rother.
Bi-layer segmentation of binocular stereo video CVPR 2005

http://research.microsoft.com/~antcrim/demos/ACriminisi_Recognition_CowDemo.wmv
Application – 3D Urban Scene Modeling

3D Urban Scene Modeling Integrating Recognition and Reconstruction, N. Cornelis, B. Leibe, K. Cornelis, L. Van Gool, IJCV 08.

http://www.vision.ee.ethz.ch/showroom/index.en.html#
Stereo systems

- Stereo vision
- Rectification
- Correspondence problem
- Active stereo vision systems
Active stereo (point)

Replace one of the two cameras by a projector
- Single camera
- Projector geometry calibrated
- What’s the advantage of having the projector? Correspondence problem solved!
Active stereo (stripe)

- Projector and camera are parallel
- Correspondence problem solved!
Laser scanning

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

Source: S. Seitz
Laser scanning

The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz
Active stereo (shadows)

J. Bouguet & P. Perona, 99

- 1 camera, 1 light source
- very cheap setup
- calibrated the light source
Active stereo (shadows)
Active stereo (color-coded stripes)

- Dense reconstruction
- Correspondence problem again
- Get around it by using color codes

L. Zhang, B. Curless, and S. M. Seitz 2002
S. Rusinkiewicz & Levoy 2002

**Rapid shape acquisition: Projector + stereo cameras**
Next lecture…

Affine Structure from Motion
Human Stereopsis


Credit slide J. Ponce
Human Stereopsis: Reconstruction

Disparity: \( d = r - l = D - F; \) \( d < 0 \)

In 3D, the horopter.

Credit slide J. Ponce
Human Stereopsis: Reconstruction

What if $F$ is not known?

Helmoltz (1909):

• There is evidence showing the vergence angles cannot be measured precisely.

• Humans get fooled by bas-relief sculptures.
Human Stereopsis: Binocular Fusion
How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one?
• There is anecdotal evidence for the latter (camouflage).

• Random dot stereograms provide an objective answer.
Issues