EECS 442 – Computer vision

Cameras

without cameras we wouldn’t have C.V.

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Reading: [FP] Chapters 1 – 3
[HZ] Chapter 6

Some slides in this lecture are courtesy to Profs. J. Ponce, S. Seitz, F-F Li
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How do we see the world?

• Let’s design a camera
  – Idea 1: put a piece of film in front of an object
  – Do we get a reasonable image?
Pinhole camera

- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**
Some history...

Milestones:
• Leonardo da Vinci (1452-1519): first record of camera *obscura*
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- Joseph Nicephore Niepce (1822): first photo - birth of photography

Photography (Niepce, “La Table Servie,” 1822)
Some history…

Milestones:
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- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière Brothers, 1895)
- Color Photography (Lumière Brothers, 1908)

Photography (Niepce, “La Table Servie,” 1822)
Let’s also not forget…

Motzu (468-376 BC)
Oldest existent book on geometry in China

Aristotle (384-322 BC)
Also: Plato, Euclid

Al-Kindi (c. 801–873)
Ibn al-Haitham (965-1040)
Pinhole camera

\[
f = \text{focal length}
\]
\[
c = \text{center of the camera}
\]
Pinhole camera

\[ P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \]

\[ \begin{cases} x' = f \frac{x}{z} \\ y' = f \frac{y}{z} \end{cases} \]

Derived using similar triangles
Pinhole camera

\[ P = [x, z] \]

\[ P' = [x', f] \]

\[ \frac{x'}{f} = \frac{x}{z} \]
Pinhole camera

Common to draw image plane *in front* of the focal point. Moving the image plane merely scales the image.

\[
\begin{align*}
x' &= f \frac{x}{z} \\
y' &= f \frac{y}{z}
\end{align*}
\]
Pinhole camera

Is the size of the aperture important?
Shrinking aperture size

- Rays are mixed up

- Why the aperture cannot be too small?
  - Less light passes through
  - Diffraction effect

Adding lenses!
Cameras & Lenses

- A lens focuses light onto the film
Cameras & Lenses

- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the focal length $f$
Cameras & Lenses

- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    [other points project to a “circle of confusion” in the image]
Cameras & Lenses

Snell’s law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]

\( \alpha_1 \) = incident angle
\( \alpha_2 \) = refraction angle
\( n_i \) = index of refraction
Thin Lenses

Snell’s law:

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]

Small angles:

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \]

- \( n_1 = n \) (lens)
- \( n_1 = 1 \) (air)

Focal length:

\[ f = \frac{R}{2(n - 1)} \]

\[ z' = f + z_o \]

\[ x' = z' \frac{x}{z} \]

\[ y' = z' \frac{y}{z} \]
Lenses are combined in various ways…

Source wikipedia
Effect of the focal length

28 mm lens

50 mm lens

70 mm lens

210 mm lens

Source: wikipedia
Dolly zooms

Exaggerated perception of depth

Compressed perception of depth
Issues with lenses: Chromatic Aberration

- Lens has different refractive indices for different wavelengths: causes color fringing

\[ f = \frac{R}{2(n-1)} \]
Issues with lenses: Radial Distortion

- Deviations are most noticeable for rays that pass through the edge of the lens

![No distortion](image1.png)

![Pin cushion](image2.png)

![Barrel (fisheye lens)](image3.png)

Image magnification decreases with distance from the optical axis
Issues with lenses: Radial Distortion

Pin cushion

Barrel (fisheye lens)
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Pinhole camera

f = focal length

\( f = \text{focal length} \)

c = center of the camera

\( c = \text{center of the camera} \)

\[
(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})
\]

\( \mathbb{R}^3 \rightarrow \mathbb{R}^2 \)
(x, y, z) → (f \frac{x}{z}, f \frac{y}{z})

Is this a linear transformation?

No — division by z is nonlinear

How to make it linear?
Homogeneous coordinates

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\]

homogeneous image coordinates

\[(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\]

homogeneous scene coordinates

• Converting *from* homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)
\]

\[
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Perspective Projection Transformation

\[
X' = \begin{bmatrix} f & x \\ f & y \\ z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]

\[
X' = M X
\]

\[
\mathbb{R}^4 \xrightarrow{H} \mathbb{R}^3
\]
From retina plane to images

Pixels, bottom-left coordinate systems
Coordinate systems
Converting to pixels

1. Off set

\[(x, y, z) \rightarrow \left( f \frac{x}{z} + c_x, f \frac{y}{z} + c_y \right)\]

\[C = [c_x, c_y]\]
Converting to pixels

1. Off set
2. From metric to pixels

\[(x, y, z) \rightarrow (f k \frac{x}{z} + c_x, f l \frac{y}{z} + c_y)\]

Units: \(k, l:\) pixel/m \\
\(f:\) m \\
\(\alpha, \beta:\) pixel \\
Non-square pixels
Converting to pixels

Matrix form?

\[
(C, x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)
\]

\[C = [c_x, c_y]\]
Camera Matrix

\[(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)\]

\[X' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\]

\[C = [c_x, c_y]\]
Camera Matrix

\[ X' = M X \]
\[ = K[I \ 0]X \]
Finite projective cameras

\[ X' = \begin{bmatrix} \alpha & s & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]

K has 5 degrees of freedom!
The mapping so far is defined within the camera reference system.

What if an object is represented in the world reference system?
3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

\[ R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \]

\[ R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \]

\[ R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
World reference system

In 4D homogeneous coordinates:

\[ X = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R \\ 0 \end{bmatrix} \begin{bmatrix} X_w \end{bmatrix} \]

\[ X' = K[I \ 0] X = K[I \ 0] \begin{bmatrix} R \ T \\ 0 \ 1 \end{bmatrix} \begin{bmatrix} X_w \end{bmatrix} \]

Internal parameters

External parameters
Projective cameras

\[ X'_3x1 = M_{3x4} \quad X = K_{3x3} \begin{bmatrix} R & T \end{bmatrix}_{3x4} \quad X_{w4x1} \]

\[ K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \]

How many degrees of freedom?

\[ 5 + 3 + 3 = 11! \]
Projective cameras

\[ X'_3 \times 1 = M X_w = K_{3 \times 3} \left[ \begin{array}{cc} R & T \end{array} \right]_{3 \times 4} X_{w4 \times 1} \quad M = \left[ \begin{array}{c} m_1 \\ m_2 \\ m_3 \end{array} \right] \]

\[ = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} X_w = \begin{bmatrix} m_1 X_w \\ m_2 X_w \\ m_3 X_w \end{bmatrix} \rightarrow \left( \frac{m_1 X_w}{m_3 X_w}, \frac{m_2 X_w}{m_3 X_w} \right) = (x, y, z)_w \]
Theorem (Faugeras, 1993)

\[
M = K[R \ T] = [K R \ K T] = [A \ b]
\]

\[
K = \begin{bmatrix}
\alpha & s & c_x \\
0 & \beta & c_y \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\alpha = f \ k;
\]

\[
\beta = f \ l
\]

\[
A = \begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
\]

- A necessary and sufficient condition for \( M \) to be a perspective projection matrix is that \( \text{Det}(A) \neq 0 \).

- A necessary and sufficient condition for \( M \) to be a zero-skew perspective projection matrix is that \( \text{Det}(A) \neq 0 \) and

\[
(a_1 \times a_3) \cdot (a_2 \times a_3) = 0.
\]

- A necessary and sufficient condition for \( M \) to be a perspective projection matrix with zero skew and unit aspect-ratio is that \( \text{Det}(A) \neq 0 \) and

\[
\begin{cases}
(a_1 \times a_3) \cdot (a_2 \times a_3) = 0, \\
(a_1 \times a_3) \cdot (a_1 \times a_3) = (a_2 \times a_3) \cdot (a_2 \times a_3).
\end{cases}
\]
Properties of Projection

- Points project to points
- Lines project to lines
- Distant objects look smaller
Properties of Projection

- Angles are not preserved
- Parallel lines meet!
Vanishing points

• Sets of parallel lines on the same plane lead to \textit{collinear} vanishing points [The line is called the \textit{horizon} for that plane]

• Each set of parallel lines meets at a different point [The \textit{vanishing point} for this direction]
One-point perspective

- Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28
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Weak perspective projection

When the relative scene depth is small compared to its distance from the camera

\[
\begin{align*}
\begin{cases}
x' &= -\frac{f'}{z} x \\
y' &= -\frac{f'}{z} y
\end{cases} & \quad \rightarrow \quad \begin{cases}
x' &= -\frac{f'}{z_0} x \\
y' &= -\frac{f'}{z_0} y
\end{cases}
\end{align*}
\]

Magnification \( m \)
Orthographic (affine) projection

Distance from center of projection to image plane is infinite

\[
\begin{align*}
  x' &= -\frac{f'}{z} x \\
  y' &= -\frac{f'}{z} y \\
  x' &= -x \\
  y' &= -y
\end{align*}
\]
Pros and Cons of These Models

• Weak perspective much simpler math.
  – Accurate when object is small and distant.
  – Most useful for recognition.

• Pinhole perspective much more accurate for scenes.
  – Used in structure from motion.
Weak perspective projection

Qingming Festival by the Riverside  Zhang Zeduan ~900 AD
The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui
HW 0.2: watch:
http://www.eecs.umich.edu/~silvio/teaching/EECS442_2009/UCSD_videos/3DVision.avi

HW 0.3: review SVD decomposition
Next lecture

• How to calibrate a camera?