

Velocity Estimation Using Optic Flow and Radar

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ABSTRACT

This paper presents the development of a static estimator for obtaining state information from optic flow and radar measurements. It is shown that estimates of translational and rotational speed can be extracted using a least squares inversion. The approach is demonstrated in a simulated three dimensional urban environment on an autonomous quadrotor micro-air-vehicle (MAV). The resulting methodology has the advantages of computation speed and simplicity, both of which are imperative for implementation on MAVs due to stringent size, weight, and power requirements.

Keywords: State estimation, Optic flow, Vision-based control, Radar, Autonomous, Micro air vehicle

1. INTRODUCTION

Unmanned air vehicles (UAVs) are a well established class of aircraft which have been in use for several decades. UAVs carry a variety of sensors capable of determining the vehicles' pose and velocity to enable a level of autonomy which includes stability control, trajectory tracking, and GPS-based waypoint navigation. However, these vehicles are designed for missions at high altitudes and are thus unable to navigate unmapped obstacles such as buildings, trees, or telephone wires. In recent years, an emphasis has been placed on the development of micro air vehicles (MAVs), a miniaturized class of UAVs whose mission profiles typically include navigating close to the ground in unmapped, cluttered outdoor or indoor environments. These vehicles require more precise sensing and control than typical UAVs to safely navigate the cluttered environments. The ability for a vehicle to autonomously estimate egomotion and proximity to obstacles is considered an advanced capability, even for large vehicles. MAVs are very small platforms, typically on the order of several hundred grams or less, so they are limited to carrying small, low weight sensors with low power and processing requirements, thus severely restricting the type of sensors and control algorithms which can be implemented onboard. As a result, the investigation of novel sensing techniques is necessary to advance MAV technology.

Naturally, vision is an appealing technique for providing a thorough knowledge of an environment. Several machine vision approaches have been investigated,¹⁻³ but many of these techniques prove to be computationally expensive and physically cumbersome, adding significant weight. However, one visual based method for detecting speed and proximity to obstacles which has proven to be viable for implementation on MAVs is optic flow. Derived from the visual perception of flying insects, optic flow is the characteristic patterns of visual motion which form on the retinas of insects as they move about an environment. These patterns are a function of relative speed and relative proximity of the insect to obstacles in the surroundings. Many studies have been conducted which investigate the use of optic flow in MAV navigation.⁴⁻⁸ These studies typically use optic flow for obstacle avoidance^{7,8} or state estimation with the additional requirement of sensors such as GPS or IMUs.⁵ Alternatively, wide field integration (WFI) of optic flow has been proven to be an effective method for obstacle avoidance and state estimation,^{4,6,9} but requires the derivation of weighting patterns based on an assumed environment structure.

The work presented here proposes an alternative method for state estimation from optic flow without the use of WFI or GPS/IMU sensors. As a function of relative speed and proximity, optic flow estimates can be merged with proximity measurements in order to extract more accurate knowledge of velocity. Consequently, introducing

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an accurate distance measurement sensor, such as radar, to a vehicle can provide proximity information which can be incorporated with optic flow measurements to obtain better speed estimates. Thus, not only is accurate knowledge of the environment obtained through optic flow and radar measurements, but accurate translational and rotational velocity information can be acquired without the use of additional sensors such as GPS or IMUs. This paper investigates a method for static estimation of translational and angular velocity of a MAV flying through a cluttered environment. A 3-D simulator developed by the Autonomous Vehicle Laboratory at the University of Maryland is utilized to test the state estimation methods presented in this paper on a quadrotor robotic platform.

This paper is organized as follows. In Section 2, a discussion of optic flow is provided and a mathematical model is presented. Section 3 derives a static estimation method for estimating translational and rotational velocity of an MAV using the optic flow model presented in Section 2, along with measurements taken by radar sensors. Section 4 validates the static estimation scheme using a simulation of a 6 degree of freedom (6-DOF) quadrotor MAV. Finally the results are discussed in Section 5, drawing the conclusion that the method of static estimation presented in this work is adequate for determining a vehicles' velocity and could be used for feedback control.

2. OPTIC FLOW MODEL

This section provides a background on optic flow sensing. The theory associated with optic flow is discussed and mathematical models are presented.

2.1 Optic Flow

Optic flow is the apparent visual motion experienced by an observer when moving through an imaged environment. True optic flow is the vector field describing the relative velocities of points within the projected image over the viewing surface, e.g. the retina of an insect. This velocity field is defined by the translational and rotational motion, as well as the relative proximity of the observer to objects in the surrounding environment. The optic flow pattern $\dot{\mathbf{Q}}$ on a spherical surface can be expressed mathematically as

$$\dot{\mathbf{Q}} = \boldsymbol{\omega} \times \mathbf{r} + \mu[\mathbf{v} - \langle \mathbf{v}, \mathbf{r} \rangle \mathbf{r}] \quad (1)$$

where $\boldsymbol{\omega} = [p, q, r]^T$ is the angular velocity, $\mathbf{v} = [u, v, w]^T$ is the translational velocity of the vantage point, and μ is the nearness function, which represents the distribution of objects in the surrounding environment. The nearness function is defined as the inverse of the distance from the observer to an object in the environment at a particular viewing angle of azimuth $\gamma \in [0, 2\pi]$ and elevation $\beta \in [0, \pi]$, thus $\mu(\gamma, \beta) = \frac{1}{d(\gamma, \beta)}$. Figure 1 illustrates the optic flow pattern from Eqn (1) can be broken down into components of azimuth and elevation:

$$\dot{\mathbf{Q}} = \dot{Q}^\gamma \hat{\mathbf{e}}_\gamma + \dot{Q}^\beta \hat{\mathbf{e}}_\beta \quad (2)$$

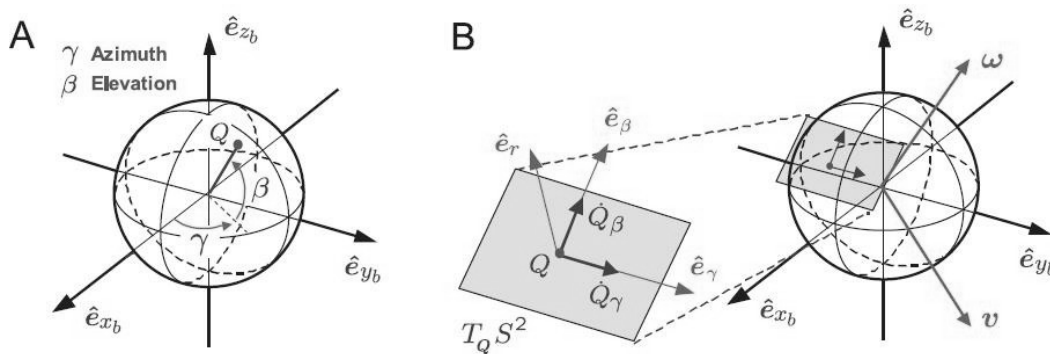


Figure 1. Geometry for spherical optic flow.

where

$$\begin{aligned}\dot{Q}^\gamma &= p \cos \beta \cos \gamma + q \cos \beta \sin \gamma - r \sin \beta + \mu(u \sin \gamma - v \cos \gamma) \\ \dot{Q}^\beta &= p \sin \gamma - q \cos \gamma + \mu(-u \cos \beta \cos \gamma - v \cos \beta \sin \gamma + w \sin \beta).\end{aligned}\quad (3)$$

In robotic applications, optic flow is measured as either 1-D flow (either $\dot{Q}^\gamma$ or $\dot{Q}^\beta$) or 2-D flow (both $\dot{Q}^\gamma$ and $\dot{Q}^\beta$). In the work presented in this research, 2-D optic flow is assumed.

3. OPTIMAL STATIC ESTIMATION OF RELATIVE STATES

Optic flow cannot be measured directly; rather, the value $\dot{\mathbf{Q}}$ is an estimate of optic flow which is dependent on the luminance incident on the imaging surface. This estimation process, along with sensor noise and contrast and texture variations throughout the surfaces in the environment, introduce error into the optic flow measurements. In addition, the radar proximity measurements are also corrupted by several sources of noise. In this section, a method of static estimation for determining translational and rotational velocity from noisy measurements is presented.

3.1 Measurement Model

The combined effect of the various uncertainties have been modeled in^{10,11} as zero mean white Gaussian noise. Thus the optic flow measurement is expressed as

$$\dot{\mathbf{Q}} = \dot{\mathbf{Q}} + \boldsymbol{\nu}, \quad (4)$$

where $\dot{\mathbf{Q}}$ is the optic flow measurement, and $\boldsymbol{\nu}$ is assumed to be zero mean, white, and uncorrelated with itself at different viewing angles. The radar distance measurements, too, are corrupted by noise, thus,

$$\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} + \mathbf{n}. \quad (5)$$

where $\tilde{\boldsymbol{\mu}}$ is the nearness measurement, and $\mathbf{n}$ is also assumed to be zero mean, white, and uncorrelated with itself at different viewing angles. The observation equation is then obtained from Eqns. (3), (4) and (5). If k discrete optic flow and radar measurements are taken on the sphere, the observation equations are then written as

$$\begin{aligned}\dot{Q}_1^\gamma &= p \cos \beta_1 \cos \gamma_1 + q \cos \beta_1 \sin \gamma_1 - r \sin \beta_1 + (\mu_1 + n_1)(u \sin \gamma_1 - v \cos \gamma_1) + \nu_1 \\ \dot{Q}_2^\gamma &= p \cos \beta_2 \cos \gamma_2 + q \cos \beta_2 \sin \gamma_2 - r \sin \beta_2 + (\mu_2 + n_2)(u \sin \gamma_2 - v \cos \gamma_2) + \nu_2 \\ &\vdots \\ \dot{Q}_k^\gamma &= p \cos \beta_k \cos \gamma_k + q \cos \beta_k \sin \gamma_k - r \sin \beta_k + (\mu_k + n_k)(u \sin \gamma_k - v \cos \gamma_k) + \nu_k \\ \dot{Q}_1^\beta &= p \sin \gamma_1 - q \cos \gamma_1 + (\mu_1 + n_1)(-u \cos \beta_1 \cos \gamma_1 - v \cos \beta_1 \sin \gamma_1 + w \sin \beta_1) + \nu_1 \\ \dot{Q}_2^\beta &= p \sin \gamma_2 - q \cos \gamma_2 + (\mu_2 + n_2)(-u \cos \beta_2 \cos \gamma_2 - v \cos \beta_2 \sin \gamma_2 + w \sin \beta_2) + \nu_2 \\ &\vdots \\ \dot{Q}_k^\beta &= p \sin \gamma_k - q \cos \gamma_k + (\mu_k + n_k)(-u \cos \beta_k \cos \gamma_k - v \cos \beta_k \sin \gamma_k + w \sin \beta_k) + \nu_k.\end{aligned}\quad (6)$$

By allowing $e_{1,j} = \nu_j + n_j(u \sin \gamma_j - v \cos \gamma_j)$ for $j = 1, 2, \dots, k$ and $e_{2,j} = \nu_j + n_j(-u \cos \beta_j \cos \gamma_j - v \cos \beta_j \sin \gamma_j + w \sin \beta_j)$ for $j = 1, 2, \dots, k$, Eqn. (6) can take the form of the linear measurement equation

$$\mathbf{z} = H\mathbf{x} + \mathbf{e}, \quad (7)$$

where $\mathbf{z}$ is the $2k \times 1$ vector of optic flow measurements $\mathbf{z} = [\dot{Q}_1^\gamma, \dot{Q}_2^\gamma, \dots, \dot{Q}_k^\gamma, \dot{Q}_1^\beta, \dot{Q}_2^\beta, \dots, \dot{Q}_k^\beta]^\top$, $\mathbf{x}$ is the 6×1 vector of angular and translational velocities $\mathbf{x} = [u, v, w, p, q, r]^\top$, $\mathbf{e}$ is the $2k \times 1$ error vector $\mathbf{e} = [\mathbf{e}_1, \mathbf{e}_2]^\top$, and H is the $2k \times 6$ matrix given by

$$H = \begin{bmatrix} \mu_1 \sin \gamma_1 & -\mu_1 \cos \gamma_1 & 0 & \cos \beta_1 \cos \gamma_1 & \cos \beta_1 \sin \gamma_1 & \sin \beta_1 \\ \mu_2 \sin \gamma_2 & -\mu_2 \cos \gamma_2 & 0 & \cos \beta_2 \cos \gamma_2 & \cos \beta_2 \sin \gamma_2 & \sin \beta_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu_k \sin \gamma_k & -\mu_k \cos \gamma_k & 0 & \cos \beta_k \cos \gamma_k & \cos \beta_k \sin \gamma_k & \sin \beta_k \\ -\mu_1 \cos \beta_1 \cos \gamma_1 & -\mu_1 \cos \beta_1 \sin \gamma_1 & \mu_1 \sin \beta_1 & \sin \gamma_1 & -\cos \gamma_1 & 0 \\ -\mu_2 \cos \beta_2 \cos \gamma_2 & -\mu_2 \cos \beta_2 \sin \gamma_2 & \mu_2 \sin \beta_2 & \sin \gamma_2 & -\cos \gamma_2 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\mu_k \cos \beta_k \cos \gamma_k & -\mu_k \cos \beta_k \sin \gamma_k & \mu_k \sin \beta_k & \sin \gamma_k & -\cos \gamma_k & 0 \end{bmatrix} \quad (8)$$

3.2 Least Squares Inversion

When written as Eqn. (7) the problem posed in the form of a standard static linear estimation problem, in which the solution of an overdetermined, inconsistent set of linear equations is sought. Gauss's principle of least squares solves for $\hat{\mathbf{x}}$, the estimate of $\mathbf{x}$ which minimizes the sum of the square of the residual errors. Thus, the goal of the least squares method is to solve for $\hat{\mathbf{x}}$ by minimizing the cost function $J = \frac{1}{2} \mathbf{e}^\top \mathbf{e}$. Substituting (7) for $\mathbf{e}$ into the equation for J , yields

$$J = J(\hat{\mathbf{x}}) = \frac{1}{2} (\tilde{\mathbf{y}}^\top \tilde{\mathbf{y}} - 2\tilde{\mathbf{y}}^\top H\hat{\mathbf{x}} + \hat{\mathbf{x}}^\top H^\top H\hat{\mathbf{x}}) \quad (9)$$

To minimize J with respect to $\hat{\mathbf{x}}$, the partial derivative is found and set equal to zero:

$$\begin{aligned} \frac{dJ}{d\hat{\mathbf{x}}} &= -\tilde{\mathbf{y}}^\top H + \hat{\mathbf{x}}^\top H^\top H \\ &= 0. \end{aligned} \quad (10)$$

Solving this equation for $\hat{\mathbf{x}}$ results in

$$\begin{aligned} H^\top H\hat{\mathbf{x}} &= H^\top \tilde{\mathbf{y}} \\ \hat{\mathbf{x}} &= (H^\top H)^{-1} H^\top \tilde{\mathbf{y}}. \end{aligned} \quad (11)$$

As long as the number of measurements k is greater than the number of unknown states n , and the measurements are linearly independent, i.e. H is full rank, Eqn. (11) provides the optimal static estimates for translational and angular velocities $\hat{\mathbf{x}} = [\hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{q}, \hat{r}]^\top$.

4. SIMULATION

The estimation methods presented in Section 3 are applied to simulations of a quadrotor vehicle flying through an urban environment, replicating the flight of an autonomous reconnaissance vehicle. This section presents the methodology and results obtained from simulation.

4.1 Methodology

The Autonomous Vehicle Laboratory at the University of Maryland developed an in-house simulation environment which provides visualization capabilities as well as the ability to compute optic flow from simulated cameras on robotic platforms. Figure 2 depicts scenes from the 3-D simulation environment. The vehicle selected for simulation is an X-UFO Quadrotor MAV made by Ascending Technologies GmbH. The quadrotor, shown in Figure 3, has an overall diameter of 40 cm, an overall mass of 505 g, and rotor diameter of 20 cm. A linearized flight dynamics model was obtained by Gremillion.¹² The kinematics and dynamics are linearized about forward flight with $u_{ref} = 1$ m/s, and are presented in Appendix A. The full nonlinear kinematic equations are used for simulation.

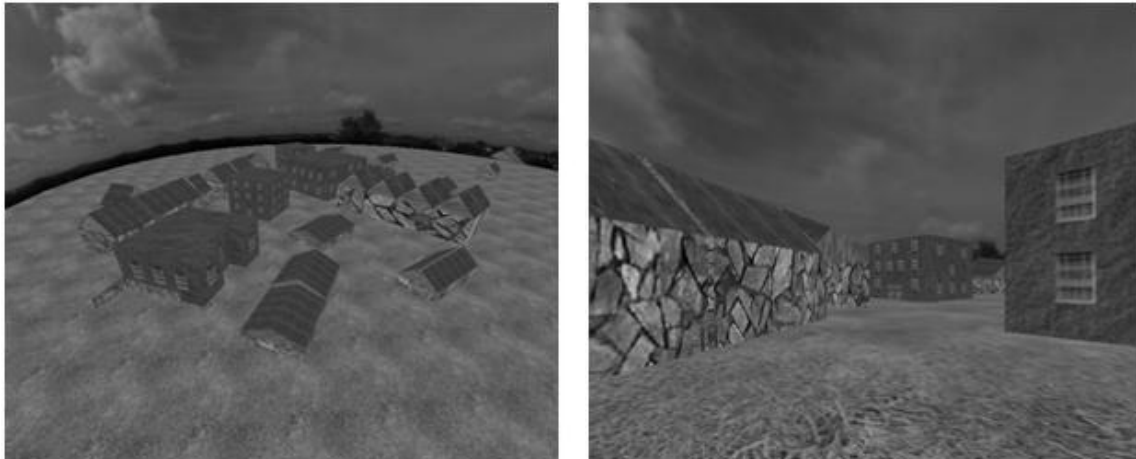


Figure 2. 3-D simulation environment.

For optic flow estimation, the virtual MAV is equipped with six cameras, each with a 90×90 deg field of view and a resolution of 128×128 pixels. The optic flow cameras cover the six sides of a cube, such that the full spherical viewing arena is imaged. However, for this work, optic flow is only measured on the bottom hemisphere, ie $0 \leq \beta \leq \frac{\pi}{2}$. In processing the images captured by the optic flow cameras, the imagery is first passed through a Gaussian blurring function to mitigate aliasing issues. A resolution iterative implementation of the Lucas-Kanade algorithm at 60 fps is implemented to calculate optic flow. During flight, 400 image points with constant angular spacing along the lower hemisphere of the uv-coordinate spherical grid are tracked. These points are mapped from a virtual sphere surface to the flat cameras via geometric projection. The objects in the simulated environment, including walls, rooftops, the ground and sky, are textured with imagery of sufficient visual contrast so that optic flow can be computed. The optic flow measurements are desampled from 400 to 100 by unweighted averaging of square groups of four adjacent nodes. To reduce noise, outlier measurements with a high final cost function or infeasibly large shift estimates are ignored in the block average.⁹

The Radiation Lab at the University of Michigan is currently developing radar sensors suitable for use on MAV platforms. The radar being developed at Michigan is a 215 GHz electronically-scanned radar with a horizontal field of view of 50 degrees, with 2-degree resolution, and a vertical field of view of 30 degrees. The range resolution is 25 cm, which is determined by the chirp bandwidth of the system, while the range of the system is approximately 200 meters, given the noise levels chosen.

In this simulation, radar measurements are simulated through distance measurements taken from the vehicle.

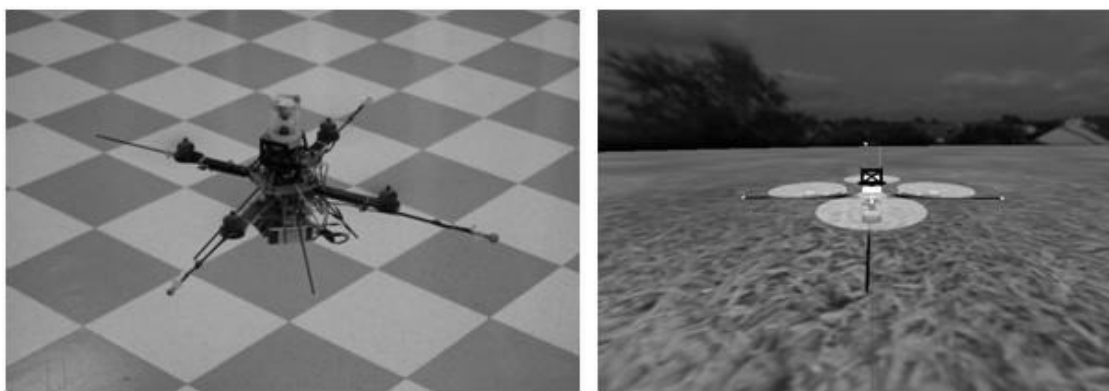


Figure 3. Actual quadrotor vs simulated quadrotor.

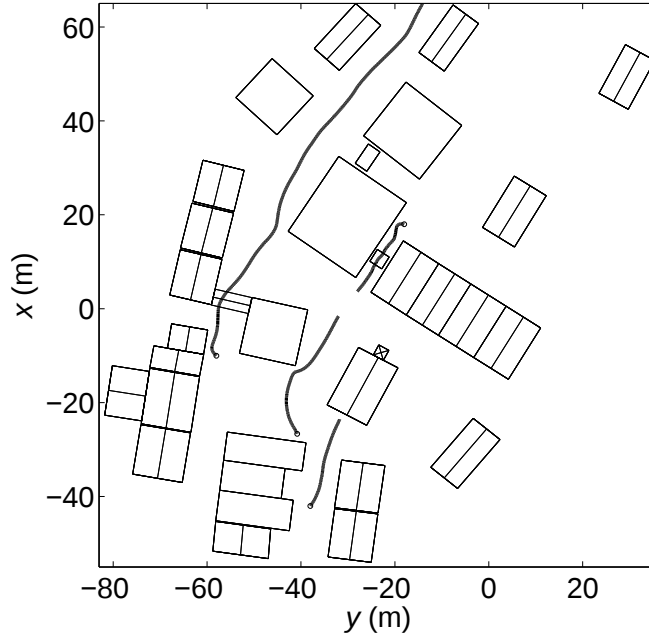


Figure 4. Quadrotor trajectories through simulated environment.

These distance measurements are individually given random, range dependent error to simulate noise in the radar sensors. For this work, 100 radar measurements are taken, corresponding to the desampled 100 optic flow measurements. Therefore, this work assumes radar sensors on the vehicle provide a 360° by 180° view spanning the bottom hemisphere of the vehicle. Based on the radar sensors in development at Michigan, this setup is slightly impractical for real-world implementation. However, the ultimate goal of the work presented here is to determine the feasibility of combining optic flow and radar for state estimation. Thus, the assumed setup is sufficient for this simulation.

4.2 Results

Figure 4 shows several of the trajectories the quadrotor followed through the simulated Fort Benning environment. The vehicle not only navigates between buildings, it also maneuvers over short obstacles, up to 1 meter high, simulating various terrain changes a reconnaissance vehicle could encounter. Figure 5 shows a sample of the results obtained during Run 1 when using the static estimation method presented in Section 3 to approximate the translational and rotational velocity of the simulated quadrotor. The results shown in this plot represent the vehicle turning away from a wall and approaching a 0.5 m tall obstacle. It is important to note that the control scheme implemented on the quadrotor during simulation not only attempts to maintain a constant forward velocity, $u_{ref} = 1$ m/s, but also maintain a constant height above the terrain of $z = 1$ m. Thus, when the vehicle approaches terrain changes, the heave velocity w and pitch rate q are affected, which explains the high frequency content observed in the heave velocity. Since the forward and lateral velocities experience slow, low frequency deviations from trim, the estimated velocities $\hat{u}$ and $\hat{v}$ are passed through a low pass filter to reduce high frequency content and smooth the data. The other states experience both high and low frequency deviations from trim, and thus cannot be filtered easily without greatly affecting the quality of the estimates.

One metric used to determine how well the estimated states match the actual values is the root-mean-square (RMS) error. The RMS error between each state, generically denoted x , is given by³

$$e_{RMS} = \sqrt{\frac{\sum_{i=1}^N (\hat{x}_i - x_i)^2}{N}}, \quad (12)$$

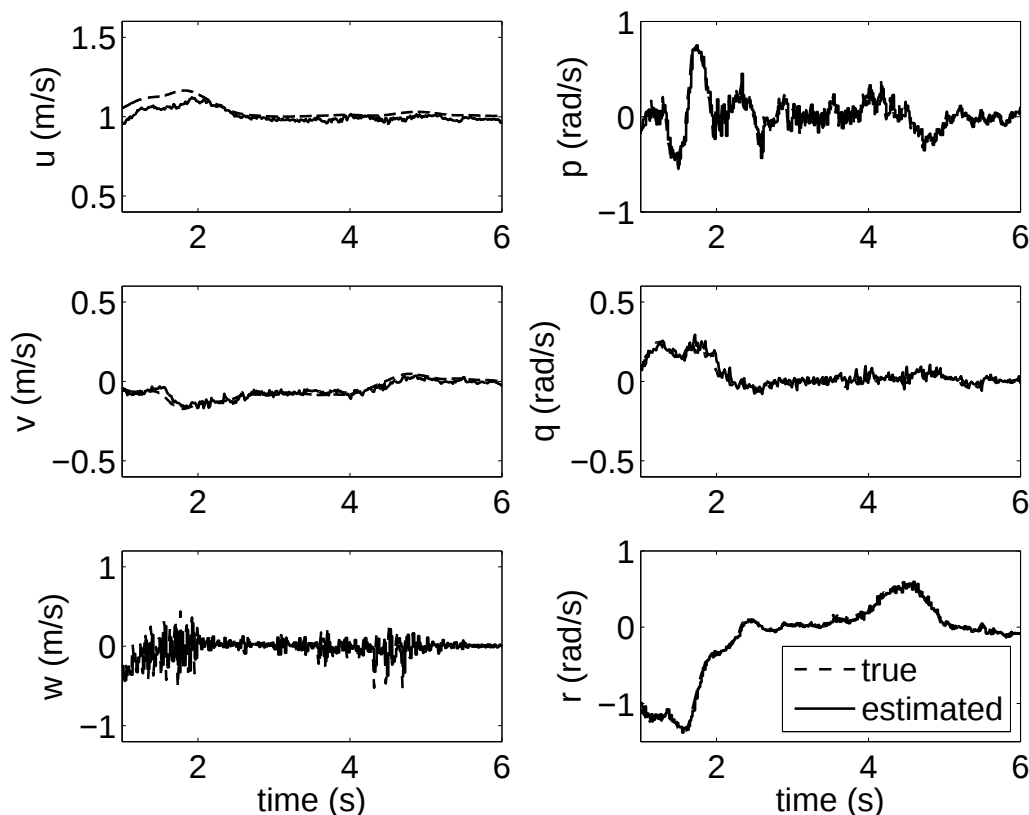


Figure 5. Estimated vs actual speeds and rates of quadrotor navigating urban environment.

where $\hat{x}_i$ is the state estimate and x_i is the true state at each time t_i , and N is the total number of samples collected. Table 1 presents the RMS errors for the estimated values of $\mathbf{x}$ for the data collected in the four trajectories shown in Figure 4.

5. DISCUSSION

Figure 5 and Table 1 both demonstrate that the estimated velocities fit the true values well. While all the states are estimated satisfactorily, particular interest is taken in how well the estimates for $\hat{u}$ and $\hat{v}$ match the true values. Traditionally, these translational velocities are difficult to measure on MAVs. Thus, the results not only validate this method of state estimation, but also demonstrate that it has great potential for use on MAV platforms. Table 1 also asserts that from the data collected in Run 1, the heave velocity has a larger RMS error than the other velocity estimates. In this run of the simulation, the vehicle approaches and traverses over a 0.5 m tall box shaped obstacle. As noted in the previous section, such a motion causes the vehicle to experience considerable heave motion as it regulates its height to safely travel over the obstacle. Thus, higher frequency

Table 1. RMS error for estimated states

State	Run 1	Run 2	Run 3	Run 4
u (m/s)	0.0787	0.0413	0.0687	0.0301
v (m/s)	0.0225	0.0151	0.0096	0.0157
w (m/s)	0.1662	0.0926	0.0896	0.0998
p (rad/s)	0.0812	0.0955	0.0671	0.0556
q (rad/s)	0.0425	0.029	0.0295	0.0258
r (rad/s)	0.0379	0.0336	0.0372	0.0271

oscillations are experienced through this section of the run. Upon closer inspection of the data, it is observed that during high frequency heave motions, the least squares estimator is able to capture the frequency well, but often does not capture the amplitude of this content as well, resulting in the larger error.

6. CONCLUSIONS

This paper demonstrates that optic flow sensing techniques can be combined with radar sensors to obtain estimates for translational and rotational velocities of a 6 DOF micro air vehicle. A method of least squares static estimation was derived using a mathematical model of 2-D optic flow to generate velocity approximations. The estimation scheme was applied to a simulation of a 6 DOF quadrotor vehicle navigating an urban environment. The results presented illustrated the viability of this estimation scheme by demonstrating an excellent fit to the true velocity values. The benefits of using the combination of optic flow and radar include the ability to obtain 1) accurate distance information from radar measurements and 2) accurate rate estimates from a simple static estimation scheme. The data collected from the combination of these sensors can allow for good obstacle detection for avoidance maneuvers as well as for velocity regulation and vehicle stabilization. Feedback control with the use of an infinite horizon linear quadratic regulation (LQR) control scheme using the state estimates obtained through the methods presented here will be investigated in future work.

APPENDIX A. QUADROTOR DYNAMICS

The dynamics equations of motion are given by

$$\begin{aligned}
 \dot{u} &= X_u u + X_\theta \theta \\
 \dot{v} &= Y_v v - u_{ref} r + Y_\phi \phi \\
 \dot{w} &= Z_w w + u_{ref} q + Z_{thr} \delta_{thr} \\
 \dot{p} &= L_p p + L_\phi \phi + L_{lat} \delta_{lat} \\
 \dot{q} &= M_q q + M_\theta \theta + M_{lon} \delta_{lon} \\
 \dot{r} &= N_r r + N_{yaw} \delta_{yaw} \\
 \dot{\phi} &= \Phi_p p + \Phi_{lat} \delta_{lat} \\
 \dot{\theta} &= \Theta_q q + \Theta_{lon} \delta_{lon} \\
 \dot{\psi} &= \Psi_r r + \Psi_{yaw} \delta_{yaw}
 \end{aligned} \tag{13}$$

The actuator saturation limits are: $|\delta_{lat}| \leq 1$, $|\delta_{lon}| \leq 1$, $|\delta_{yaw}| \leq 1$, $|\delta_{thr}| \leq 1$. The characteristic stability derivatives are defined in Table 2.

Table 2. Quadrotor Parameter Values

Parameter	Value	Parameter	Value
X_u	-0.27996	Φ_p	0.9655
Y_v	-0.22566	Θ_q	0.9634
Z_w	-1.2991	Ψ_r	0.6748
L_p	-2.5110	Z_{thr}	-39.282
M_q	-2.4467	L_{lat}	11.468
N_r	-0.4948	M_{lon}	9.5711
X_θ	-10.067	N_{yaw}	3.5647
Y_ϕ	9.8648	Φ_{lat}	0.0744
L_ϕ	-21.358	Θ_{lon}	0.0594
M_θ	-18.664	Ψ_{yaw}	0.0397

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