New twists on eigen-analysis (or spectral) learning

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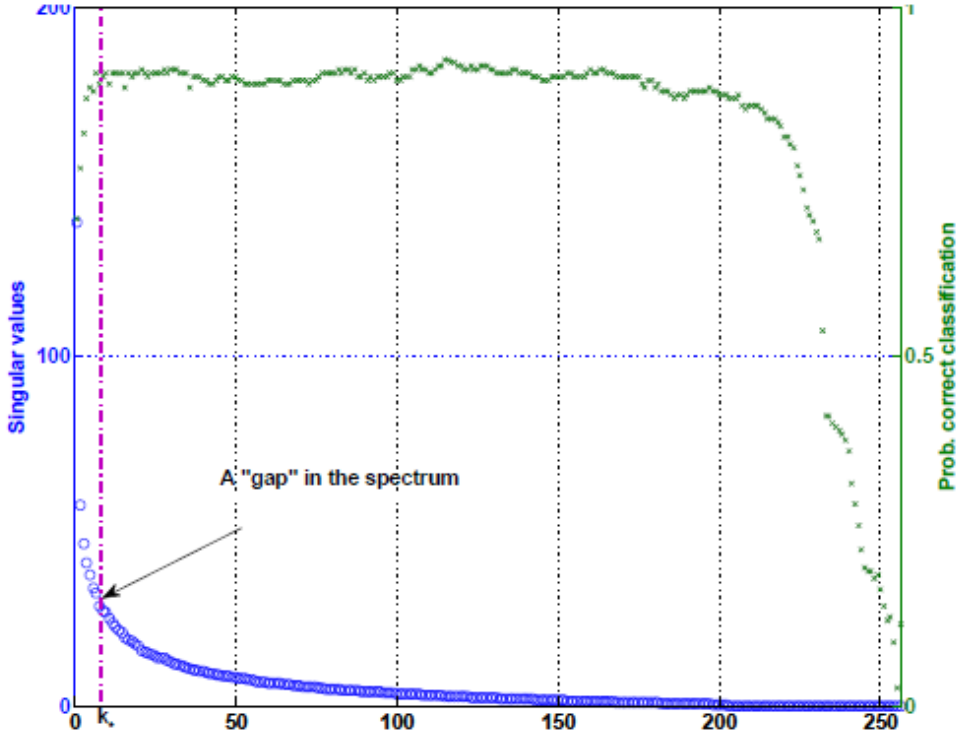
Role of eigen-analysis in Data Mining

- Prinicipal Component Analysis
- Latent Semantic Indexing
- Canonical Correlation Analysis
- Linear Discriminant Analysis
- Multidimensional Scaling
- Spectral Clustering
- Matrix Completion
- Kernalized variants of above

- Eigen-analysis synonymous with Spectral Dim. Red.

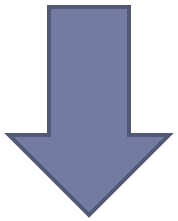
Mechanics of Dim. Reduction



- Many heuristics for picking dimension
  - "Play-it-safe-and-overestimate" heuristic
  - "Gap" heuristic
  - "Percentage-of-explained-variance" heuristic

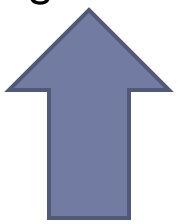
Motivation for this talk

- Large Matrix Valued Dataset Setting:
  - High-Dimensional Latent Signal Variable + Noise



- "Out intuition in higher dimensions isn't worth a damn"

George Dantzig, MS Mathematics, 1938 U. of Michigan



Random matrix theory = Science of eigen-analysis

New Twists on Spectral learning

- 1) All (estimated) subspaces are not created equal
- 2) Value to judicious dimension reduction
- 3) Adding more data can degrade performance

- Incorporated into next gen. spectral algorithms
  - Improved, data-driven performance!
  - Match or improve on state-of-the-art non-spectral techniques

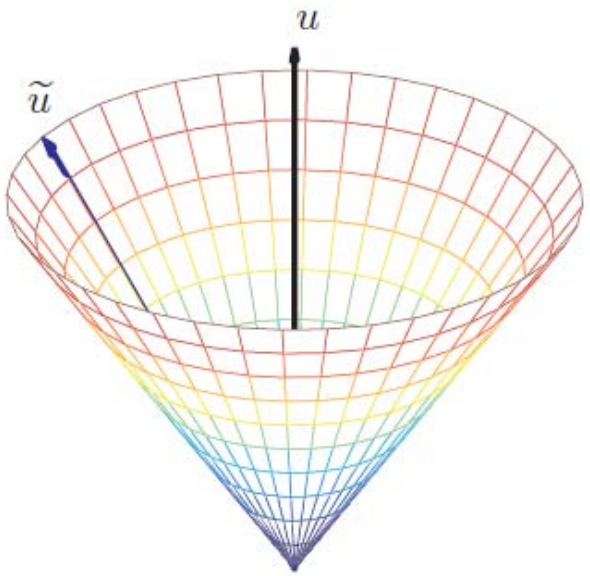
Analytical model

$$\widetilde{X}_n = \sum_{i=1}^{k} \theta_i u_i v_i' + X_n$$

- Low dimensional (= k) latent signal model
- X_n is an n x m Gaussian "noise-only" matrix
- c = n/m = # rows / # columns of data set
- Theta ~ SNR

1) All estimated subspaces are not equal

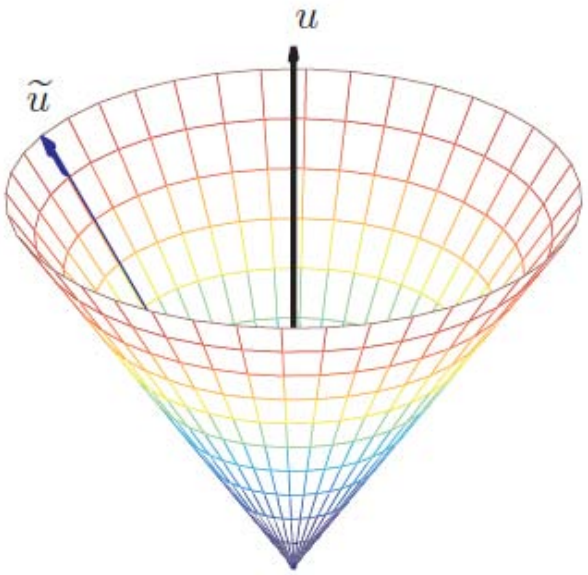
$$|\langle \widetilde{u}_i, u_i \rangle|^2 = \frac{\theta_i^4 - c}{\theta_i^4 + c\,\theta_i^2} + o(1)$$

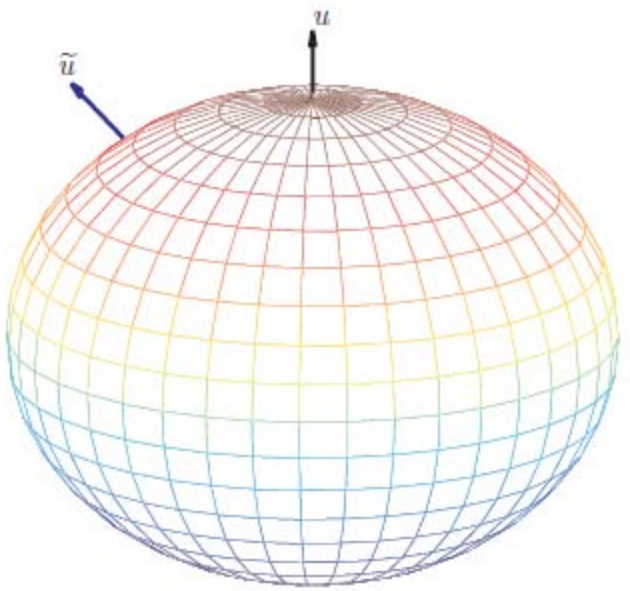


- c = # rows / # columns in data set
- Theta ~ SNR
- Subspace estimates are biased (in geometric sense above)

2) Value of judicious dim. reduction

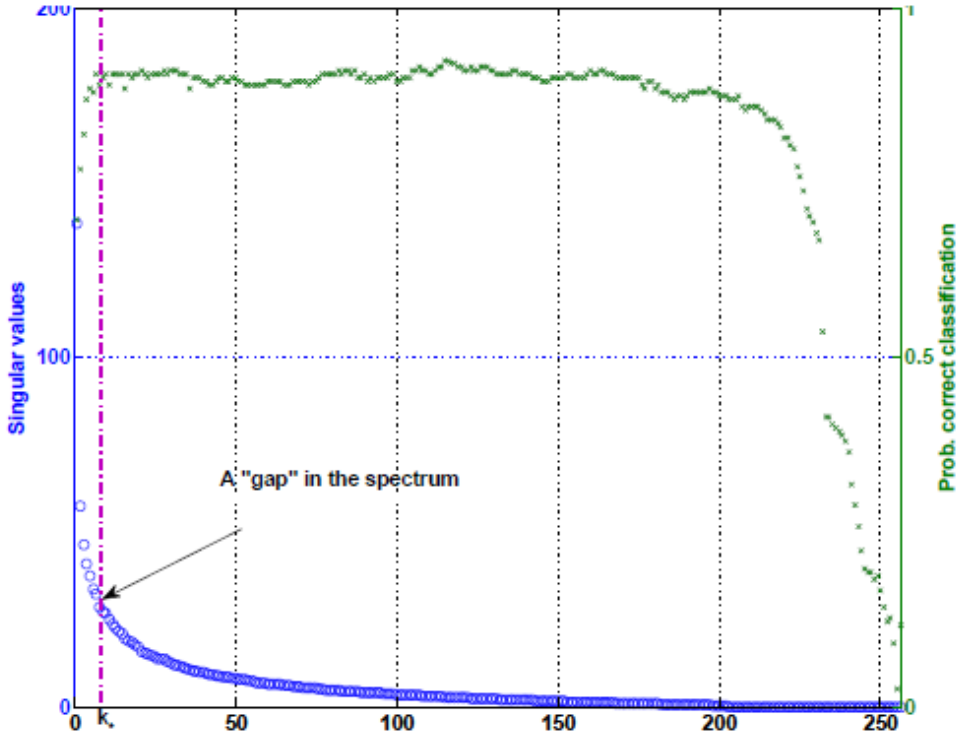
$$|\langle \widetilde{u}_i, u_i \rangle|^2 = \begin{cases} \dfrac{\theta_i^4 - c}{\theta_i^4 + c\,\theta_i^2} + o(1) & \text{if } \theta_i \geq c^{1/4} \\ o(1) & \text{otherwise.} \end{cases}$$





- "Playing-it-safe" heuristic injects additional noise!

Mechanics of Dim. Reduction



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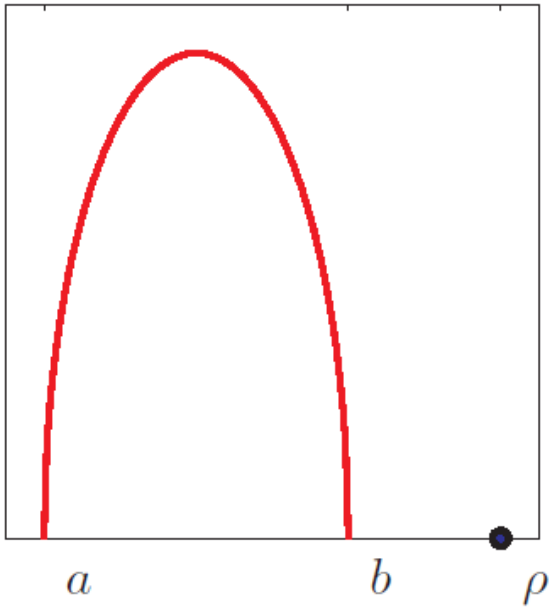
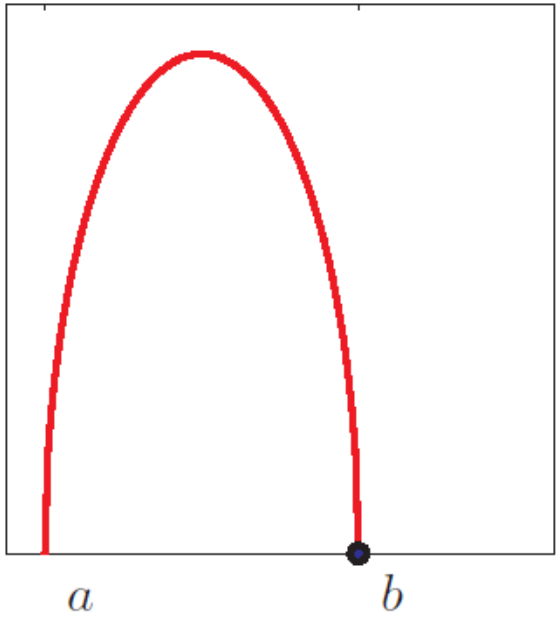
  - "Play-it-safe-and-overestimate" heuristic
  - "Gap" heuristic
  - "Percentage-of-explained-variance" heuristic

What about the gap heuristic?



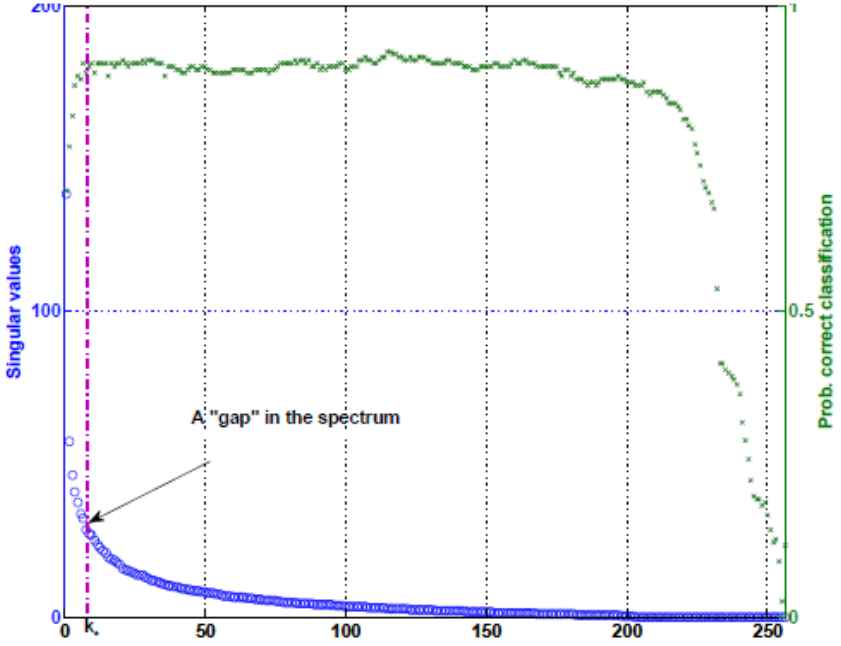
(a) Eigenvalue: $\theta > \theta_c$

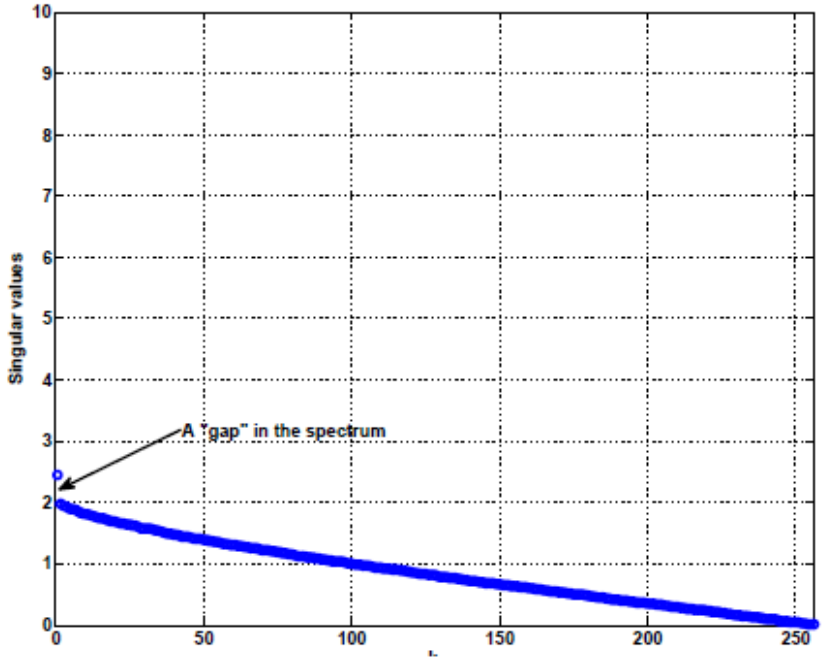


(c) Eigenvalue: $\theta \leq \theta_c$

- No "gap" at breakdown point

Percentage-of-variance heuristic?





- O(1) eigenvalues that look "continuous" are noise!
  - Including those dimensions injects noise!
  - Value of judicious dimension reduction!

3) More data can degrade performance

$$|\langle \widetilde{u}_i, u_i \rangle|^2 = \begin{cases} \dfrac{\theta_i^4 - c}{\theta_i^4 + c\,\theta_i^2} + o(1) & \text{if } \theta_i \geq c^{1/4} \\ o(1) & \text{otherwise.} \end{cases}$$

- c = n/m = # rows / # columns
- Consider n = m so c = 1
  - n' = 2n, m' = m
  - New critical value = $2^{1/4}$ x Old critical value!
  - Weaker latent signals now buried!
  - Value to adding "correlated" data and vice versa!

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