Analyzing Security Protocols using Probabilistic I/O Automata

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References

- Authors: Ran Canetti, Ling Cheung, Dilsun Kaynar, Moses Liskov, Nancy Lynch, Olivier Pereira, Roberto Segala
- Using Probabilistic I/O Automata to Analyze an Oblivious Transfer Protocol. MIT CSAIL-TR-2005-1001, August ‘05.
- Revision, CSAIL-TR-2006-046, June ‘06.
- Task-Structured Probabilistic I/O Automata. WODES ’06.
- Full version in progress.
- Using Task-Structured Probabilistic I/O Automata to Analyze an Oblivious Transfer Protocol. CSAIL-TR-2006-047, June ‘06.
- Time-Bounded Task-PIOAs: A Framework for Analyzing Security Protocols. DISC ’06.

General goals

- Develop techniques for
  - Modeling security protocols precisely.
  - Proving their correctness rigorously.
- Techniques should handle both functional correctness and security properties.
- Should be able to describe cryptographic primitives, computational limitations.
- Tractable, usable methods.

I/O Automata models and methods

- Our favorite tools for modeling, analyzing distributed algorithms, communication protocols, safety-critical systems,…
- Describe systems using:
  - I/O Automata (IOA) **[Lynch, Tuttle]**
  - Timed I/O Automata (TIOA) **[Kaynar, Lynch, Segala, Vaandrager]**
  - Hybrid I/O Automata (HIOA) **[Lynch, Segala, Vaandrager]**
  - Probabilistic I/O Automata (PIOA) **[Segala, Lynch]**
- Prove correctness using:
  - Compositional methods: Infer properties of a system from properties of its pieces.
  - Invariant assertions: Properties that hold in all reachable system states.
  - Simulation relations: Relate system descriptions at different levels of abstraction.

Uses of I/O Automata

- Basic I/O Automata:
  - Basic distributed algorithms: Consensus, mutual exclusion, spanning trees,…
- Timed I/O Automata:
  - Communication protocols.
  - Timing-sensitive distributed algorithms.
  - Simple hybrid systems.
- Hybrid I/O Automata:
  - More complex hybrid systems (controlled vehicles, aircraft).
  - Mobile ad hoc networks.
- Probabilistic I/O Automata:
  - Randomized distributed algorithms.

- So, they should work for Security Protocols too!

An early attempt [Lynch, CSFW 99]

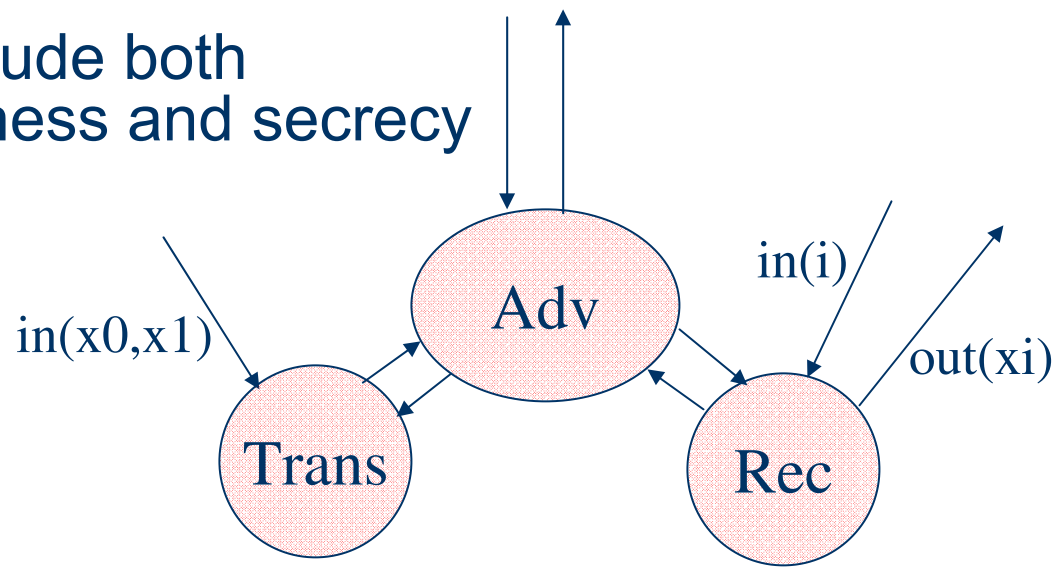
- I/O Automata models and proofs of shared-key communication systems
- Models:
  - Diffie-Hellman key distribution protocol, and
  - Shared-key communication protocol that uses the keys.
- Proves correctness and secrecy for the complete system, using a composition theorem.
- No probabilities used here---just plain I/O Automata.
- Treats the cryptosystem formally (algebraically) not computationally.

Limitations of this approach

- Doesn't describe some key features of cryptographic protocols:
  - Computational limitations.
  - Probabilistic behavior.
  - Small probabilities of guessing secret information.
  - "Knowledge" of a fact (rather than a value).
- So, we decided we needed a modeling framework that supports these features too.

Specific goal

- Prove correctness of a simple 2-party Oblivious Transfer protocol [Goldreich, Micali, Wigderson 87].
- Oblivious Transfer requirements:
  - Transmitter gets input bits, x0 and x1, from the "Environment".
  - Receiver gets input bit i, an index it uses to select an input.
  - Receiver should output only the chosen Transmitter input xi.
  - Adversary (who hears all communication) shouldn't learn anything.
- Requirements include both functional correctness and secrecy properties.

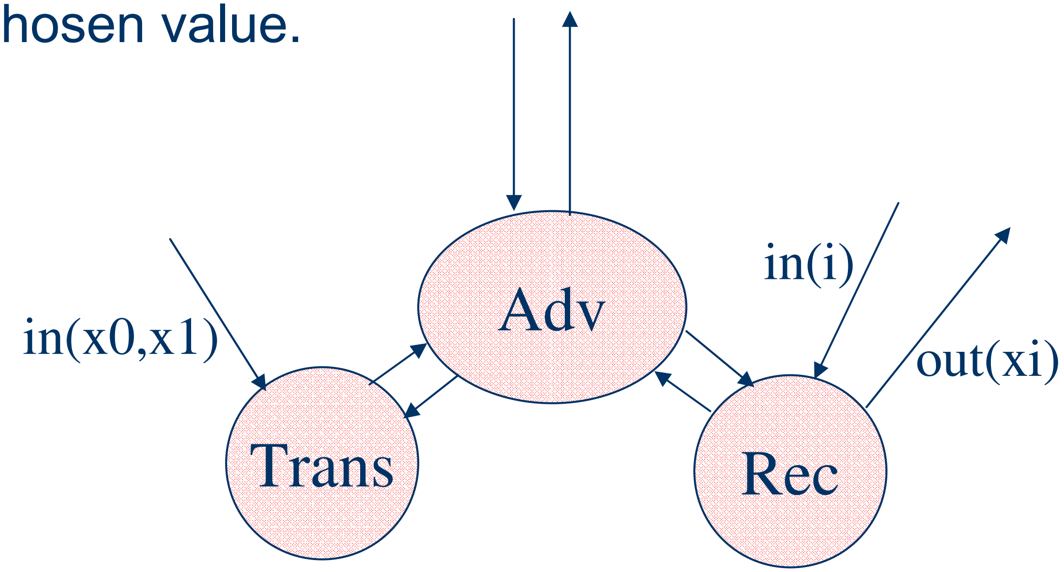


Four versions of the problem

- Depending on whether Transmitter and/or Receiver is corrupted.
- Adversary also sees inputs, outputs, random choices of corrupted parties.
- But it should not learn anything else.

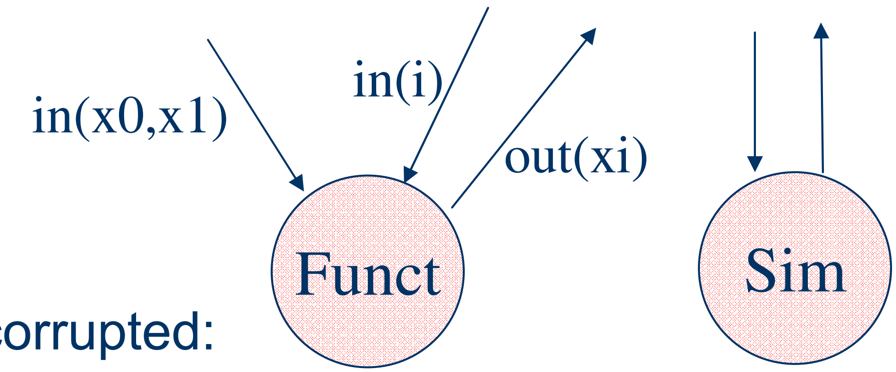
Oblivious Transfer Protocol

- Trans chooses a random trap-door permutation f, sends to Rec.
- Rec chooses random numbers y0 and y1, computes f(yi), where i is its input, keeps y(1-i) unchanged, sends results to Trans.
- Trans applies f^{-1} to both, extracts hard-core bits, xors them with its inputs x0 and x1, sends results back to Rec.
- Rec decodes the chosen value.

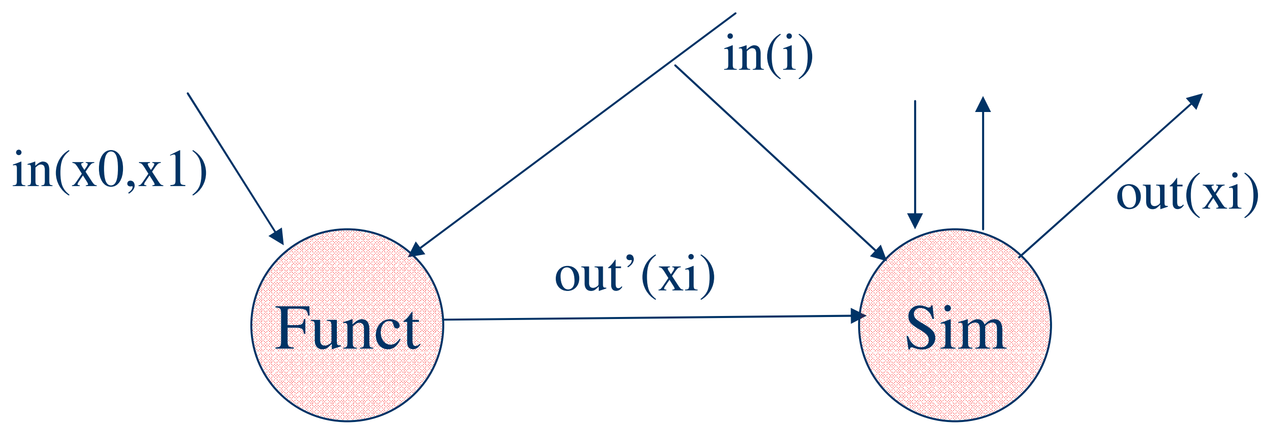


PIOA modeling

- Use PIOAs to model both protocol and requirements specification.
- E.g. Specification, when no one is corrupted:

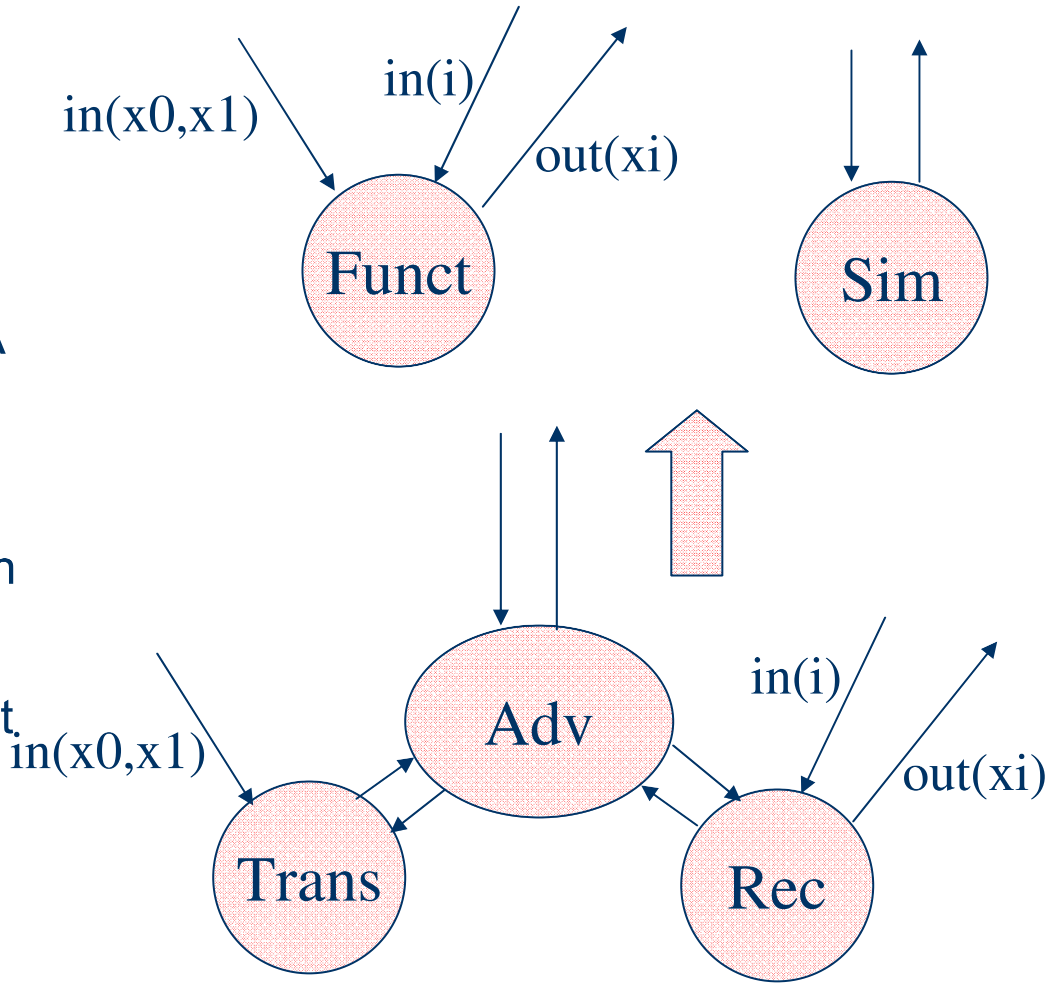


- Specification, when Rec is corrupted:



Showing the protocol "implements" the specification system:

- Uses new implementation notion, $\leq_{neg,pt}$.
- Relates Protocol PIOA system to Specification PIOA system.
- "For every poly-time-bounded environment PIOA E, every probabilistic execution of Protocol + E yields "approximately the same" external behavior as some probabilistic execution of Spec + E."
- Approximation: Negligible difference in probability that E "accepts".
- Expresses functional correctness and secrecy.



Proving correctness

- Break the proof into several stages, using system descriptions at several levels of abstraction.
- Prove some stages using PIOA simulation relations.
  - Assertions relating states of the two PIOAs.
  - Use a new type of simulation relation, more general than previous PIOA simulation relations.
  - Can express complex correspondences between random choices at different levels.
  - These prove not just $\leq_{neg,pt}$, but stronger $\leq_0$.
- Other stages involve secrecy aspects of a cryptographic primitive (a trap-door function).
  - Proofs adapted from computational crypto “Distinguisher” arguments.
  - Usually proved by contradiction.
  - We recast in terms of mappings between PIOAs, without contradictions.

What's new here?

- Modeling everything, in complete detail, using PIOAs:
  - Protocols.
  - Requirements, including functionality and secrecy.
- Proofs using simulation relations, in several stages.
  - Some stages use PIOA simulation relations.
  - Other stages express Distinguisher arguments.
  - Separate different types of reasoning.
- New PIOA theory:
  - Task-PIOAs, for resolving scheduling nondeterminism.
  - A new kind of simulation relation.
  - A way to express poly-time computation restrictions.
- New ways of expressing computational crypto reasoning:
  - Redefine cryptographic primitives using $\leq_{neg,pt}$.
  - Infer $\leq_{neg,pt}$ for systems that use the primitives.

Related work

- **[Segala 95]** Probabilistic I/O Automata theory
- **[Canetti 01]** Universally Composable (UC) security
- **[Pfitzmann, Waidner 01], [Backes, Pfitzmann, Waidner 04]** Composable security
- **[Mitchell et al.]** Modeling/analyzing security protocols using process algebras, with probabilistic poly-time processes.
- **[Shoup 04]** Cryptography proof sketches using many levels of abstraction ("games").
- Work on protocol proofs using formal cryptography, e.g., **[Dolev, Yao 83], [Lynch 99].**
- Work on protocol proofs using computational cryptography.
- Work relating the two, e.g. **[Abadi, Rogaway 02], [Canetti, Herzog 05].**

Talk Outline:

1. Overview (done)
2. Task-PIOAs
    1. PIOAs (review)
    2. Task-PIOA definitions
    3. New simulation relation
    4. Adding computational limitations
3. Oblivious Transfer Modeling and Analysis
    1. Specification model
    2. Protocol model
    3. Correctness theorems
    4. Modeling the cryptographic primitives
    5. Correctness proof
4. Conclusions

2.1. PIOAs [Segala]

A PIOA P consists of:

- Q: a countable set of states,
- q: a start state,
- I, O, and H: countable sets of input, output, and internal actions
- D, a transition relation---a set of triples of the form
  (state, action, probability measure on states).

Axioms:

- Input-enabling
- Next-transition determinism

PIOAs can make both

- Nondeterministic choices (next action), and
- Probabilistic choices (next state).

Closed PIOA: No input actions.

PIOAs

- Scheduler for a PIOA P:
  - Chooses the next action (fine-grained control).
  - Choice may depend on entire prior history (full information).
- PIOA + Scheduler yield:
  - Probabilistic execution
  - Trace distribution (probability measure on sequences of external actions)
- Operations:
  - Composition
  - Hiding (of output actions)
- Simulation relation notion
  - Relates states to distributions on states.
  - Implies inclusion of sets of trace distributions.

PIOAs

- Traditional PIOA schedulers are too powerful for the security protocol setting:
  - Choose the next action (fine-grained control).
  - Choice may depend on entire prior history (full information).
- In particular, scheduling choices may depend on secret information, supposedly hidden in the states of non-corrupted protocol participants.
- Scheduler can "leak" secret information to adversarial parties, by encoding it in the choices of scheduled actions.
- So, we defined more restricted, partial-information "task schedulers".

2.2. Task-PIOA definitions

- Task-PIOA T = (P,R)
  - PIOA + equivalence relation on output and internal actions.
  - Task = equivalence class of actions
    - E.g., “send” actions for round 1 messages.
  - Action determinism: At most 1 action in each task enabled in each state.
- Task schedule: Arbitrary sequence of tasks.
  - Models an oblivious task scheduler.
  - Does not depend on dynamic information generated during execution.
- Applying a task schedule to the initial state:
  - Resolves all nondeterminism.
  - Yields unique probabilistic execution, unique trace distribution.
- More generally, we can applying a task schedule to:
  - A probability distribution on states, or even
  - A probability distribution on finite executions.

Task-PIOA operations

- Composition:
  - Compose the PIOAs.
  - Take the union of the sets of tasks.
- Hiding (of actions).

Task-PIOA implementation relation

- Environment E for T:
  - Task-PIOA that "closes" T.
  - Has special "accept" output action.
    - Used to express E's distinguishing power.
- $T_1 \leq_0 T_2$:
  - For every Environment PIOA E for both T_1 and T_2, every trace distribution of $T_1 \| E$ (obtained from any task schedule) is also a trace distribution of $T_2 \| E$ (obtained from some task schedule).

Task-PIOA Compositionality

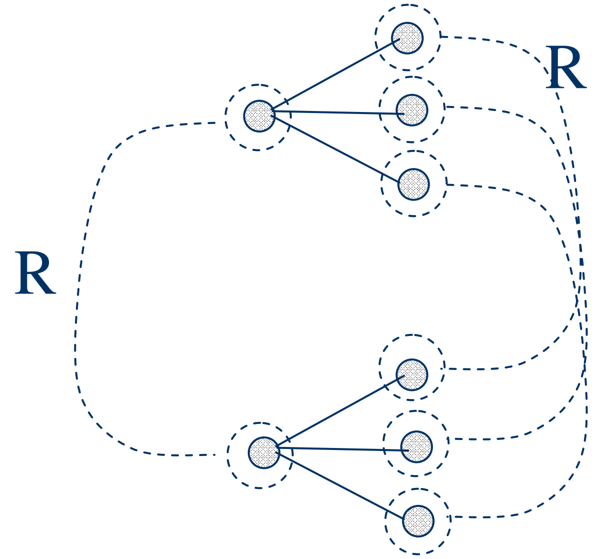
- Theorem: If $T_1 \leq_0 T_2$ then $T_1 \| T_3 \leq_0 T_2 \| T_3$.
- Proof: Straightforward, because of the way the implementation notion $\leq_0$ is defined (in terms of mappings from environments to sets of trace distributions).

2.3. New kind of simulation relation

- For comparable, closed task-PIOAs T_1 and T_2.
- ε_1 R ε_2, for probability measures ε_1 and ε_2 on finite execs of T_1 and T_2 with the same trace distribution.
- Uses expansion operator Exp(R) on such relations R.
- Two conditions (slightly simplified):
  - Start condition: Start states of T_1 and T_2 are R-related.
  - Step condition: There is a mapping c from tasks of T_1 to finite sequences of tasks of T_2 such that, if ε_1 R ε_2 and t is a task of T_1, then apply(ε_1,t) Exp(R) apply(ε_2,c(t)).
- A bit more general than "apply(ε_1,t) R apply(ε_2,c(t))".
- Soundness: Every trace distribution of T_1 is a trace distribution of T_2.

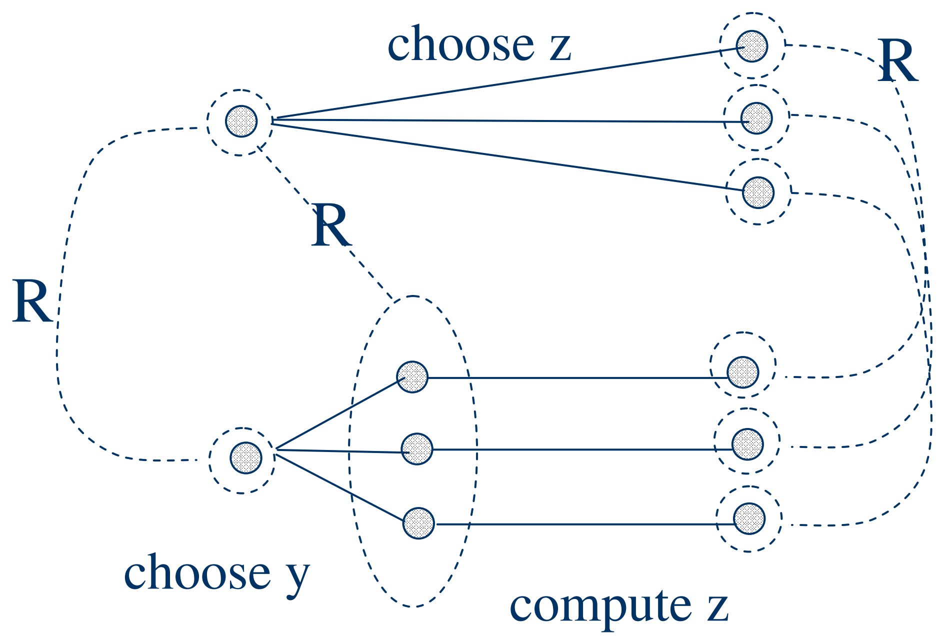
New simulation relation

- Flexible.
- Allows us to relate individual results of random choices at two levels:



New simulation relation

- Allows us to relate random choices made at different times at the two levels.



Soundness of simulation relations

- Theorem: Every trace distribution of T_1 is also a trace distribution of T_2.
- Proof: By a somewhat involved inductive argument.

2.4. Adding computational limitations

- b-time-bounded task-PIOA (b a constant):
  - States, actions, transitions, etc., have bit-string representations, length ≤ b, identifiable in time ≤ b.
  - Time ≤ b to determine next action, next state.
- b-time-bounded task schedule:
  - At most b tasks in the sequence.
- Extend to indexed families of tasks and task schedules, where b is a function of the index.

New notion of implementation

- $T_1 \leq_{neg,pt} T_2$:
  - "For every poly-time-bounded Environment E for both T_1 and T_2, every trace distribution of $T_1 \,||\, E$ (with any poly-time task schedule) is approximately the same as some trace distribution of $T_2 \,||\, E$ (with some poly-time task schedule)."
  - "Approximately the same": Difference in probability that E outputs "accept" is negligible
- $\leq_{neg,pt}$ transitive.
- $\leq_{neg,pt}$ preserved by composition.

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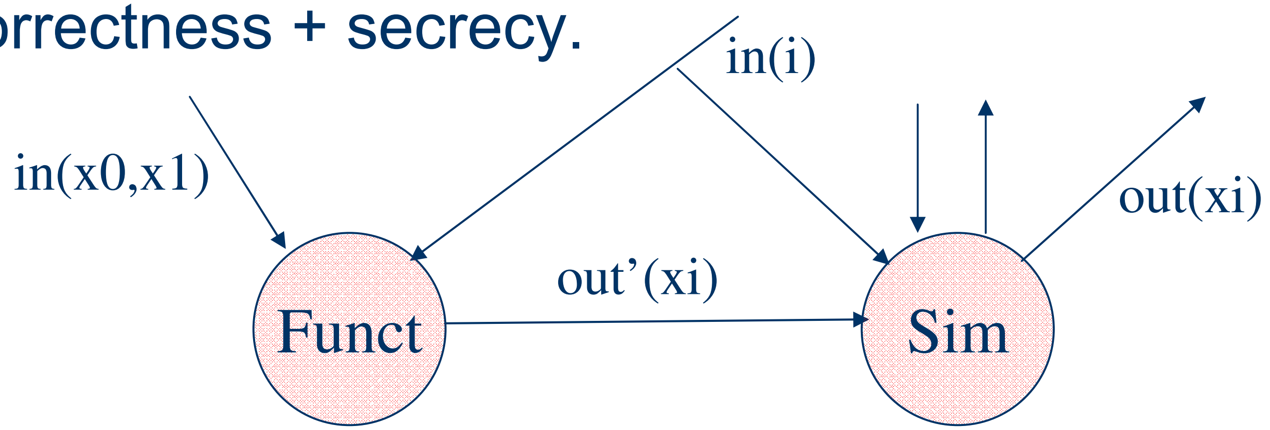
3.1. Specification: Receiver corrupted

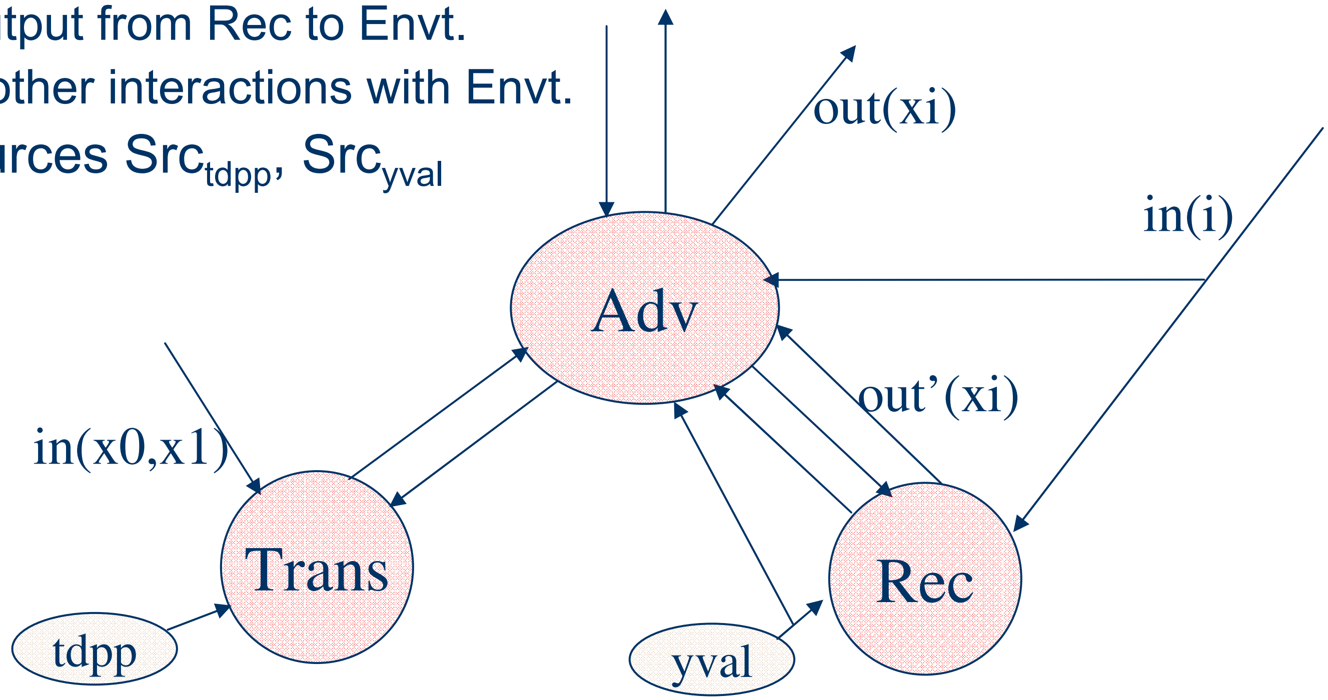
- Funct:
  - State: inval(Trans), inval(Rec)
  - Transitions: Record inputs, output inval(Trans)(inval(Rec))
- Functional correctness + secrecy.



- Sim:
  - Sees Rec input.
  - Gets output (the chosen input) from Funct; relays to environment.
  - Arbitrary other interactions with environment.
  - Doesn't learn (or reveal) non-chosen input.

3.2. OT protocol model: Rec corrupted

- Trans, Rec
- Adversary communication service:
  - Can eavesdrop, delay, reorder, drop messages.
  - Sees Rec inputs.
  - Relays output from Rec to Envt.
  - Arbitrary other interactions with Envt.
- Random sources Src_{tdpp}, Src_{yval}



OT Protocol, informal description

On inputs (x0,x1) for Trans, i for Rec:

Trans chooses a random trap-door permutation f: D → D, sends f to Rec.

Rec chooses two random elements, y0, y1 in D, computes zi = f(yi), z(1-i) = y(1-i), sends (z0,z1) to Trans.

Trans computes b0 = B(f^{-1}(z0)) xor x0, b1 = B(f^{-1}(z1)) xor x1, sends (b0,b1) to Receiver.

Receiver outputs B(yi) xor bi, which is equal to xi.

Trans Task-PIOA

- State: inval, tdpp, zval, bval
- Transitions:
  - Record inval, tdpp inputs
  - send(1,f):
    - Precondition: f = tdpp.funct
  - Record zval received in message 2.
  - fix-bval:
    - Precondition: tdpp, zval, inval defined
    - Effect: bval(0) := B(tdpp.inverse(zval(0))) xor inval(0); bval(1) := B(tdpp.inverse(zval(1))) xor inval(1)
  - send(3,b):
    - Precondition: b = bval

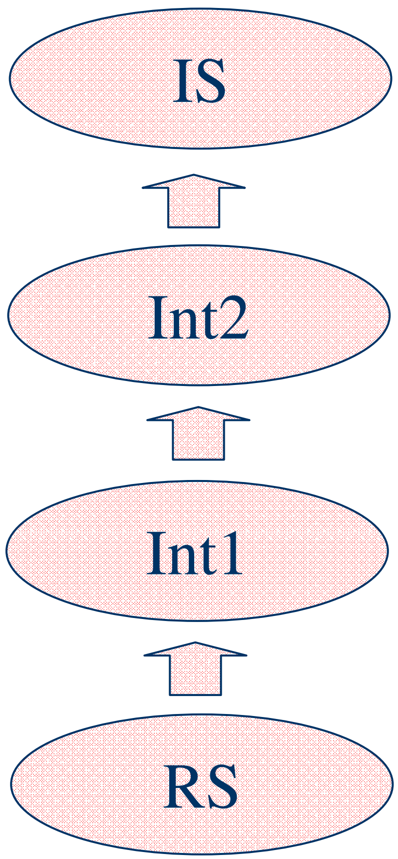
Rec Task-PIOA

- State: inval, tdp, yval, zval, outval
- Transitions:
  - Record inval, yval inputs
  - Record tdp received in message 1.
  - fix-zval:
    - Precondition: tdp, yval, inval defined
    - Effect: zval(inval) := tdp(yval(inval));
      zval(1-inval) := yval(1-inval)
  - send(2,z)
    - Precondition: z = zval
  - receive(3,b):
    - Effect: If yval defined then outval := b(inval) xor B(yval(inval))
  - out(x):
    - Precondition: x = outval

3.3. Correctness Theorems

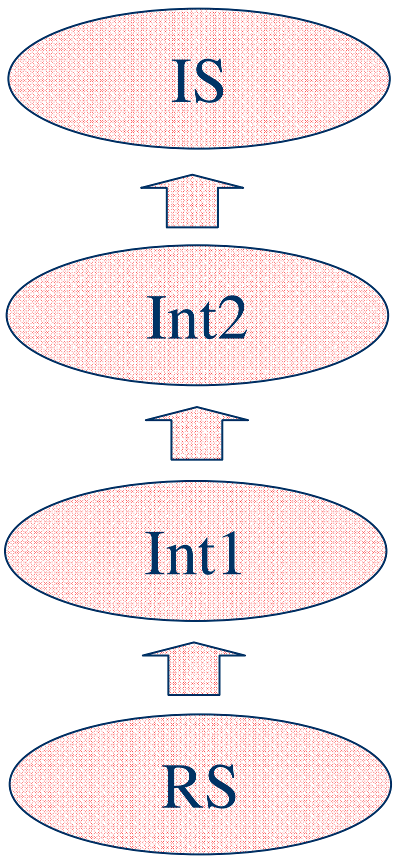
- Four cases, based on which parties are corrupted.
- Theorem: If RS ("Real System") is a family of OT protocol systems in which the family of Adv components is poly-time-bounded, then there is a family IS ("Ideal System") of OT requirements systems in which the family of Sim components is poly-time-bounded, and such that RS $\leq_{neg,pt}$ IS.
- Consider case where Rec is corrupted.
- Proof: Uses four levels of abstraction:

Levels-of-abstraction proof



- Sim component of IS:
  - Calculates bval for Rec's chosen index using xor of hard-core bit and Trans input.
  - Obtains the Trans input from Funct.
  - For non-chosen index, chooses bval randomly.
- Int2 calculates non-chosen bval using xor of random bit and Trans input.
- Int1 similar, but calculates non-chosen bval using xor of hard-core bit and Trans input.

Levels-of-abstraction proof



- Top and bottom mappings are simulation relations (of our new kind).
- Mapping from Int1 to Int2 is different:
  - The two levels are identical, except that Int1 calculates the non-chosen bval using xor of hard-core bit and Trans input, while Int2 uses random bit and Trans input.
  - Difference: Hard-core bit vs. random bit.
  - Here we use the definition of a hard-core bit, and a cryptographic Distinguisher argument.

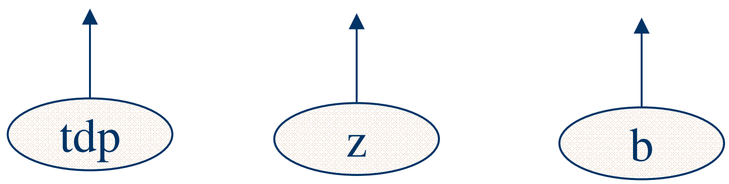
3.4. Modeling the crypto primitives

- Trap-door permutation and inverse.
- Hard-core predicate.
- Traditional definition: B is a hard-core predicate for domain D if for every polynomial-time-computable predicate G, the following two experiments output 1 with probabilities that differ by a negligible (sub-inverse-polynomial) function:
  - Experiment 1:
    - Choose random trap-door permutation f.
    - Choose random y in D.
    - Output G(f,f(y),B(y))
  - Experiment 2:
    - Choose random f, y as above.
    - Choose random bit b.
    - Output G(f,f(y),b).

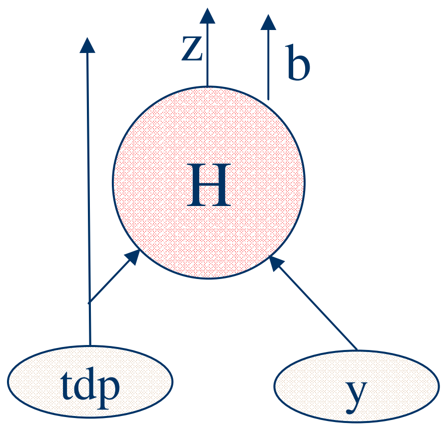
Reformulated in terms of Task-PIOAs

B is a hard-core predicate for D if SH(B) $\leq_{neg,pt}$ SHR, where:

- SHR: Three random sources:



- SH(B): Two random sources and a hard-core automaton H:



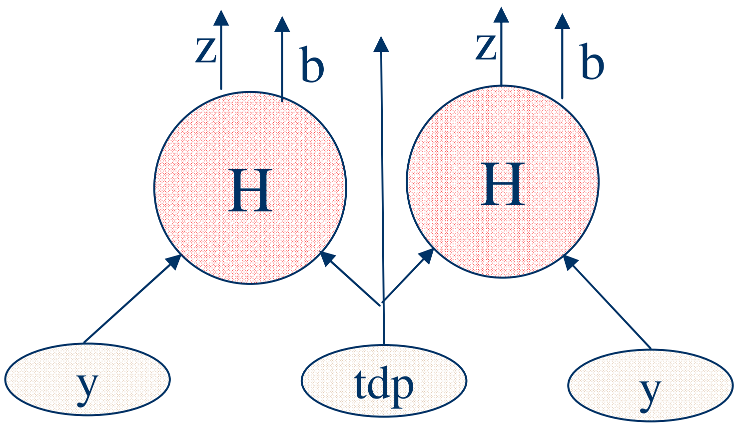
H computes tdp(y) and B(y)

Equivalence of the two definitions

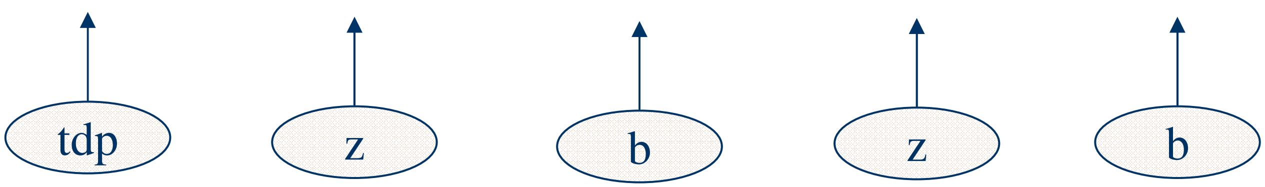
- Theorem: B is a hard-core predicate for D according to the traditional definition, if and only if B is a hard-core predicate according to the new, task-PIOA-based definition.
- Nice, because it lets us apply composition theorems for task-PIOAs to obtain results about systems that use a hard-core predicate.

Example theorem about use of hard-core predicates

- Can use a hard-core predicate twice:

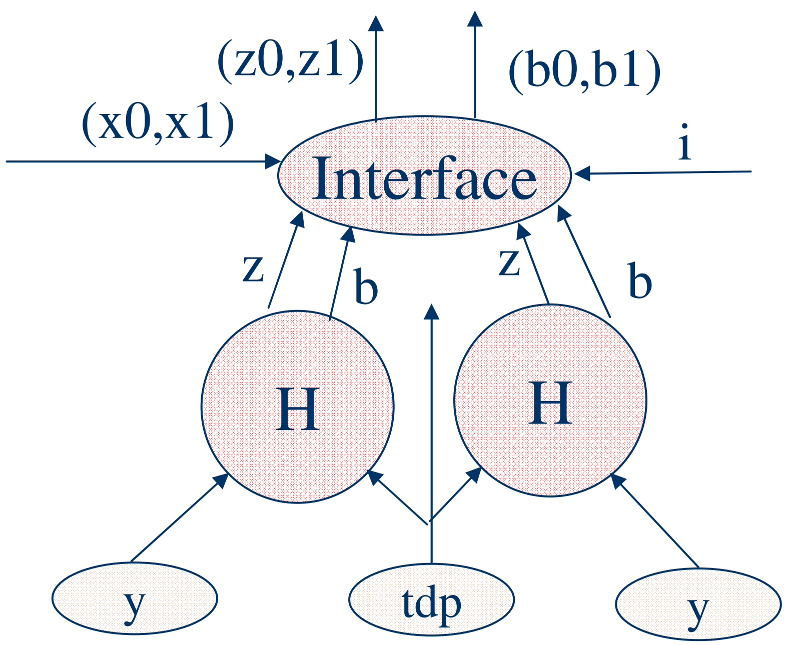


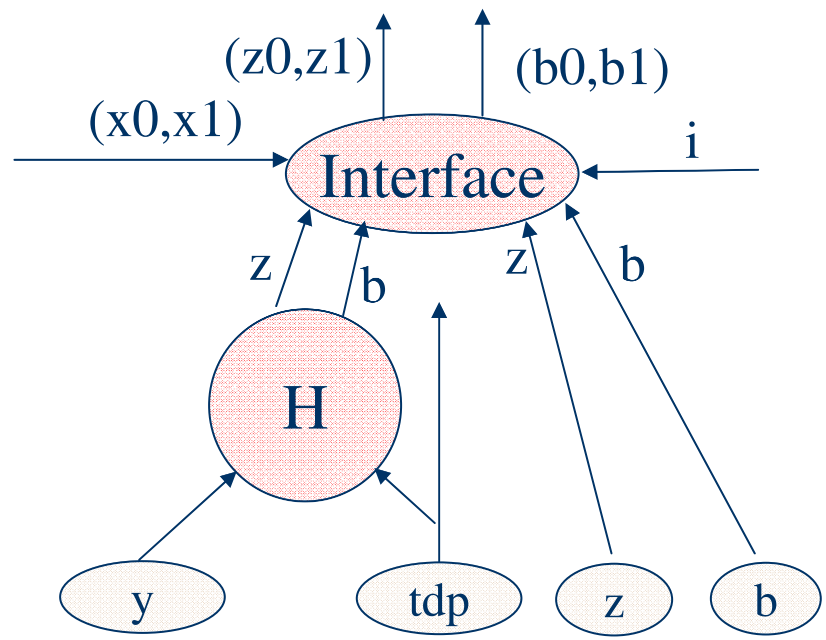
- And it implements five random sources:



Theorem used to show Int1 $\leq_{neg,pt}$ Int2

- Interface xors two hard-core bits with input values.
- Implements Interface composed with one H and random sources.
- Similar to Int1 and Int2 systems.





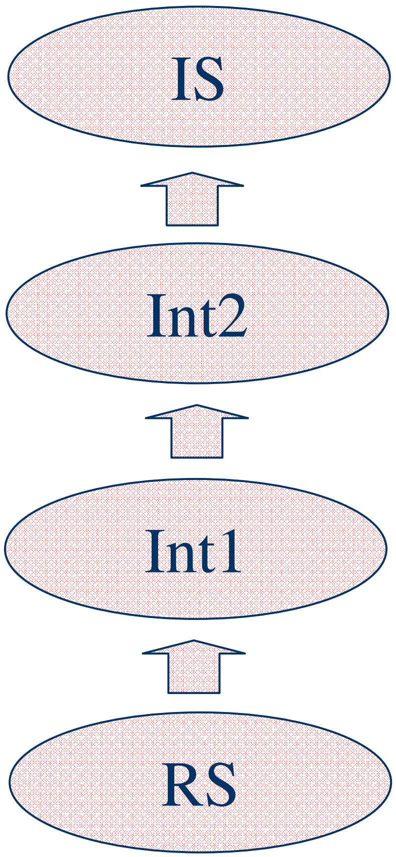
Theorems about hard-core predicates

- Describe various ways in which hard-core bits can be incorporated into a system.
- Infer that the system implements ($\leq_{neg,pt}$), a similar system using random bits.
- Implementation results follow from:
  - New definition of hard-core predicate.
  - General task-PIOA composition theorems, saying that $\leq_{neg,pt}$ is preserved by composition.

3.5. Correctness proof, revisited

- Recall the main theorem and proof outline.
- Four cases, based on which parties are corrupted.
- For each case, show Theorem:
  - If RS is a family of OT protocol systems in which the family of Adv components is poly-time-bounded, then there is a family IS of OT spec systems in which the family of Sim components is poly-time-bounded, and such that RS $\leq_{neg,pt}$ IS.
- Consider case where Rec is corrupted.
- Proof: Four levels of abstraction.

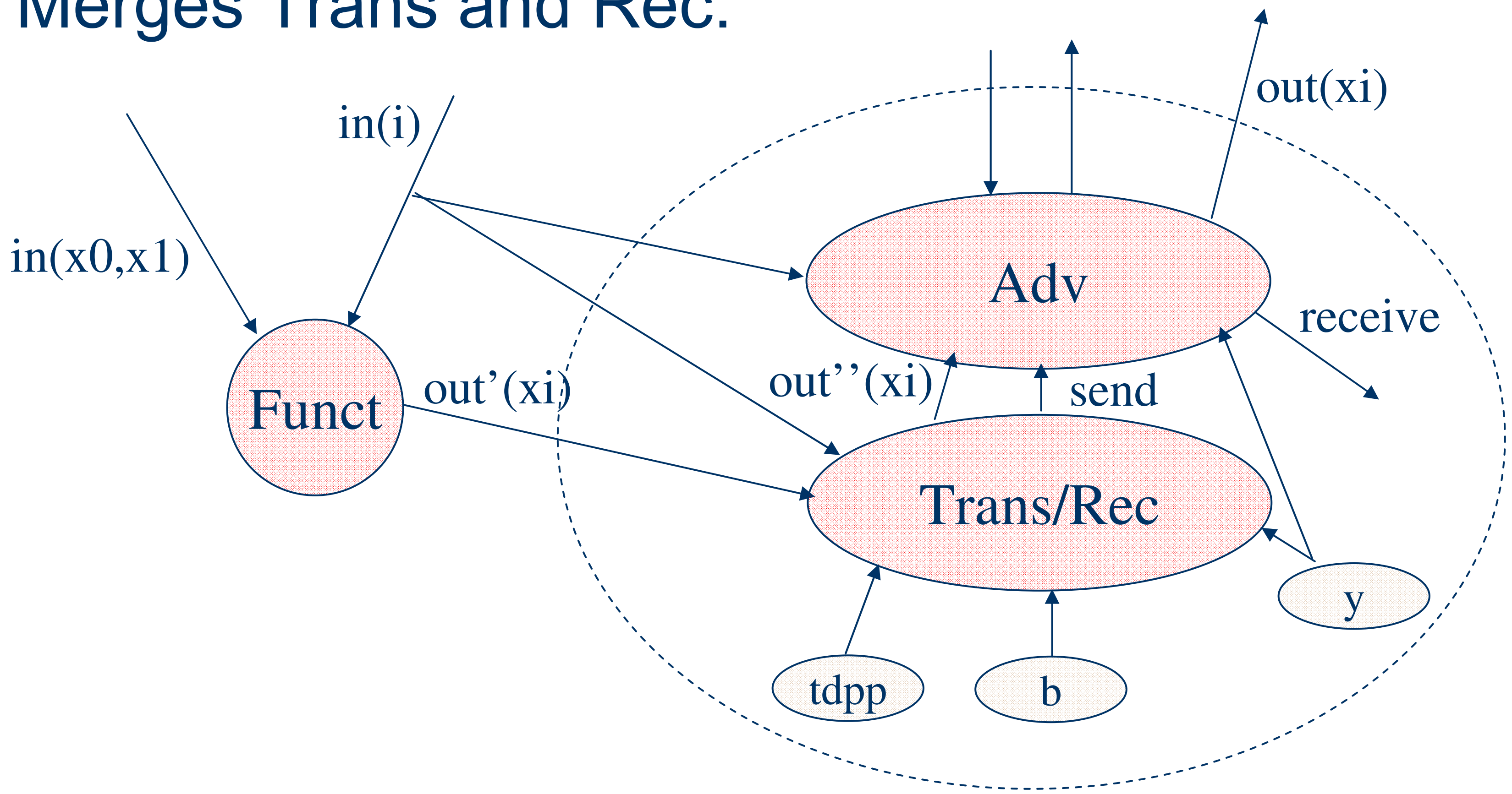
Proof: Rec corrupted



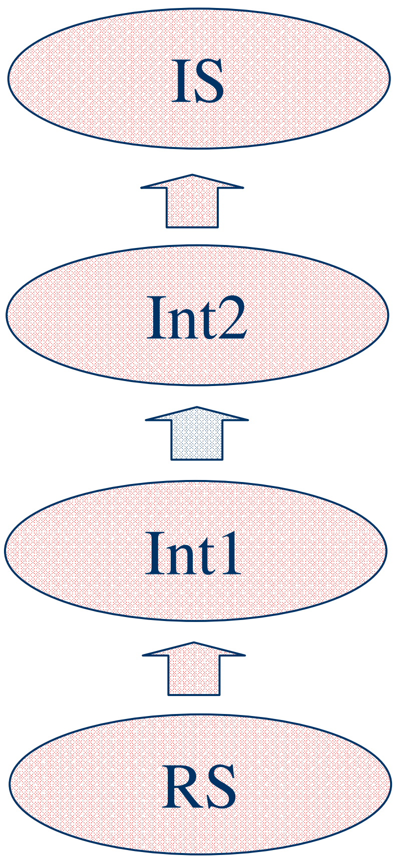
- Show $\leq_{neg,pt}$ for all stages, combine using transitivity.
- Sim component of IS:
  - Calculates bval for chosen index using xor of hard-core bit and Trans input.
  - Obtains Trans input from Funct output.
  - For non-chosen index, uses random bit.
- Int2 similar, calculates non-chosen bval using xor of random bit and Trans input.
- Int1 similar, calculates non-chosen bval using xor of hard-core bit and Trans input.
- Interesting reasoning about cryptographic primitives is confined to the proof relating Int1 and Int2.

Sim component of IS

- Merges Trans and Rec:



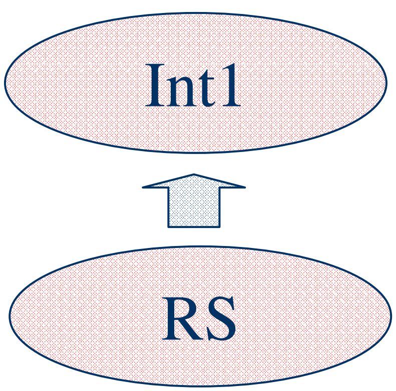
Int1 $\leq_{neg,pt}$ Int2

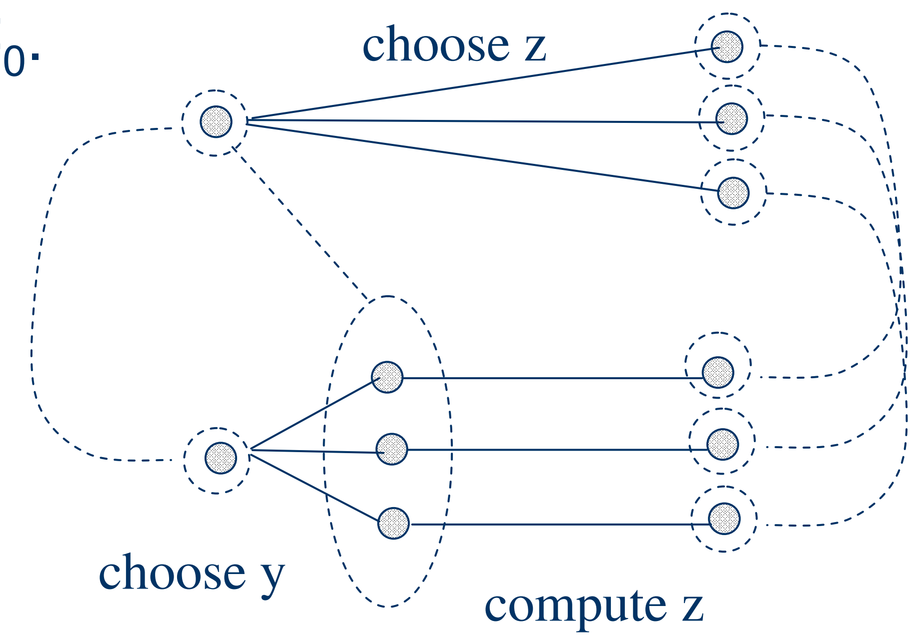


- Int1 and Int2 are identical, except:
  - Int1 calculates bval for non-chosen index using xor of hard-core bit and Trans input.
  - Int2 uses random bit and Trans input.
- Both systems use hard-core bits for bval(chosen index).
- Difference: Hard-core vs. random bit.
- Correspondence follows from previous theorem about using hard-core bits.

RS $\leq_{neg,pt}$ Int1

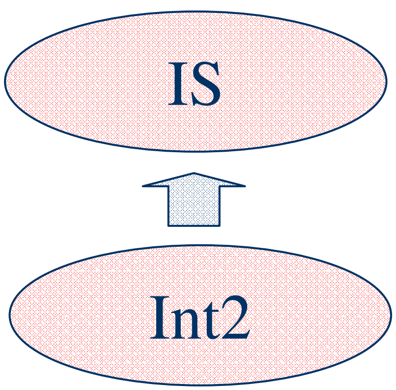
- Compose both with arbitrary Environment E.
- Simulation relation.
- Discrepancy:
  - In RS, yvals are chosen randomly, then zvals computed.
  - In Int1, zvals are chosen randomly.
- Shows stronger relation $\leq_0$.





Int2 $\leq_{neg,pt}$ IS

- Compose both with arbitrary Environment E.
- Simulation relation.
- Discrepancy:
  - In IS, bval for non-chosen index is chosen randomly.
  - In Int2, cval is chosen randomly, then xor'ed with Trans input.
  - Either way, they are random values.
- Shows stronger relation $\leq_0$.



Talk Outline:

1. Overview (done)
2. Task-PIOAs (done)
   1. PIOAs (review)
   2. Task-PIOA definitions
   3. New simulation relation
   4. Adding computational limitations
3. Oblivious Transfer Modeling and Analysis (done)
   1. Specification model
   2. Protocol model
   3. Correctness theorems
   4. Modeling the cryptographic primitives
   5. Correctness proof
4. Conclusions

Summary

- Developed techniques for modeling and analyzing security protocols, based on the PIOA modeling framework.
- Used them to carry out a formal proof for [GMW87] OT protocol.
- Required us to extend PIOAs to Task-PIOAs:
  - New partial-information scheduling mechanism (task schedules).
  - Implementation relation $\leq_0$.
  - Composition theorem.
  - New kind of simulation relation, proved to be sound for $\leq_0$.
- Time-bounded PIOAs.
  - Approximate, time-bounded implementation relation $\leq_{neg,pt}$.
  - Composition theorem.
  - Used to express security protocol correctness.
  - Used to model cryptographic primitives' secrecy properties.

Oblivious Transfer models

- Specification model:
  - Expresses both functional correctness and secrecy.
  - Formulated in terms of $\leq_{neg,pt}$.
  - Style similar to [Canetti] Universal Composability (UC) and [Backes, Pfitzmann, Waidner] universal reactive simulatability.
- Protocol model:
  - Transmitter, Receiver,
  - Adversary communication system, can eavesdrop, delay, lose, reorder messages.

Oblivious Transfer proofs

- Multi-stage mapping proofs, using $\leq_{neg,pt}$.
- Include computational cryptography issues:
  - Time-bound restrictions on adversaries, environments.
  - Crypto primitives (trap-door function, hard-core bit).
  - Distinguisher arguments, reformulated.
- Computational cryptography reasoning isolated to one stage, which uses a task-PIOA redefinition of the cryptographic primitives.

Evaluation

- Usable, scalable methods for carrying out complete, rigorous proofs of security protocols.
- Proofs decompose into manageable pieces.
  – Different pieces show different kinds of properties, using different kinds of reasoning
- Inductive, assertional methods.
- Combines nicely with formal cryptographic proofs.

Future work

- Apply methods to more security protocols:
  - More complex protocols.
  - More powerful adversaries.
- Cryptographic primitives:
  - Redefine other cryptographic primitives in terms of $\leq_{neg,pt}$.
  - Prove results about their use in protocols.
  - Reformulate traditional Distinguisher arguments using $\leq_{neg,pt}$.
- Precise comparison with related approaches, e.g. [Backes, Pfitzmann, Waidner], and [Mitchell, et al.]
- General results, e.g., about protocol composition, standard classes of adversaries.

Thank you!